Math 201 Midterm Exam 1 — Mar. 10, 2017

Name: ______________________________

Section Leader (Circle one): S. Harrop T. Ren E. Lee

Section Time (Circle one): T 3:00-3:50 T 4:30-5:20 Th 1:30-2:20 Th 3:00-3:50

B. Tzolova H. Koh T. Clingman C. Zhang

T 1:30-2:20 T 3:00-3:50 Th 3:00-3:50 Th 4:30-5:20 T 4:30-5:20

- Complete the following problems. In order to receive full credit, please make sure to *justify your answers.* You are free to use results from class or the course textbook as long as you clearly state what you are citing.
- **You have 50 minutes.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted (nor are they needed). If you finish early, you must hand your exam paper to a proctor.
- Please check that your copy of this exam contains 8 numbered pages and is correctly stapled.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

The following boxes are strictly for grading purposes. Please do not mark.

Question	Points	Score
1	15	
2	25	
3	20	
4	15	
5	25	
Total	100	

1. (15 points) Use Gauss-Jordan elimination to compute rref(A) for $A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 4 \end{bmatrix}$.

2. Consider the matrix $A = \begin{bmatrix} 0 & 1 & -1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) (5 points) Find *one* solution to $A\vec{x} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$.

(b) (5 points) Find a vector $\vec{y}$ so that the system $A\vec{x} = \vec{y}$ has *no* solutions.

(c) (5 points) Determine $\operatorname{null}(A) := \dim \ker(A)$.

(d) (10 points) Recall, $A = \begin{bmatrix} 0 & 1 & -1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Determine a basis of $\ker(A)$.

3. Let $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$. Let $A^2 = A \cdot A$, $A^3 = A \cdot A^2$ and I_3 denote the 3×3 identity matrix.

(a) (10 points) Verify that $A^3 + A = 2I_3$.

(b) (10 points) Explain why A is invertible and determine its inverse.

4. (15 points) Let $T : \mathbb{R}^m \to \mathbb{R}^m$ and $S : \mathbb{R}^m \to \mathbb{R}^n$ be linear transformations. Define $H : \mathbb{R}^m \to \mathbb{R}^n$ by

$$H(\vec{x}) = S(T(\vec{x}) + 2\vec{x}).$$

Verify that H is a linear transform and determine $[H]$, the standard matrix of H, in terms of $[T]$ and $[S]$, the standard matrices of T and S.

5. (a) (5 points) State what it means for $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^n$ to be linearly independent.

(b) (10 points) Suppose $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^n$ are linearly independent. Show that

$$\vec{w}_1 = \vec{v}_1 + \vec{v}_2, \quad \vec{w}_2 = \vec{v}_2 + \vec{v}_3, \quad \vec{w}_3 = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$$

are linearly independent.

(c) (10 points) Suppose that $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^n$. Show that the vectors

$$\vec{w}_1 = \vec{v}_1 - \vec{v}_2 - \vec{v}_3, \quad \vec{w}_2 = 2\vec{v}_1 - 3\vec{v}_2 - \vec{v}_3, \quad \vec{w}_3 = \vec{v}_3 - \vec{v}_2$$

are *not* linearly independent.