THE JOHNS HOPKINS UNIVERSITY
Krieger School of Arts and Sciences
SECOND MIDTERM EXAM - FALL 2005
110.201 – LINEAR ALGEBRA

Instructor: Professor Carel Faber
Duration: 50 minutes November 22, 2005

No calculators allowed **Total = 100 points**

NAME: Carel Faber

SECTION (weekday and time): 1, 2, 3, 4

ETHICS PLEDGE:

I agree to complete this examination without unauthorized assistance from any person, materials, or device.

SIGNATURE: DATE:

1. [**25** points] Let P_1 be the linear space of polynomials $f(t)$ of degree ≤ 1. Let T from P_1 to P_1 be the linear transformation given by

$$T(-1) = -5 - 2t \qquad \text{and} \qquad T(1+2t) = -3.$$

(a) [**7** points] Find the matrix A of T with respect to the standard basis $\mathcal{A} = (1, t)$.

(b) [**8** points] Find the matrix B of T with respect to the basis $\mathcal{B} = (1+t, 2+t)$.

(c) [**5** points] Find the change of basis matrix S from the basis $\mathcal{B}$ to the basis $\mathcal{A}$.

(d) [**5** points] Is SBS^{-1} equal to A? Motivate your answer.

2. [**25** points] Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 7 & 0 \\ 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 3 & 4 & 5 & 2 & 6 \end{bmatrix}.$$

3. [**25** points] Consider the linear space P_1 of polynomials of degree ≤ 1 with inner product

$$\langle f, g\rangle = \int_0^1 f(t)g(t)dt.$$

(a) [**5** points] Determine the norm of the element $f(t) = 1$ of P_1.

(b) [**5** points] Show that $g(t) = 2t - 1$ is orthogonal to $f(t)$.

(c) [**5** points] Determine the norm of the element $q(t)$ of P_1.

(d) [**10** points] Find the linear polynomial $k(t) = a + bt$ that best approximates the function $h(t) = t^2 - t$ on the interval $[0, 1]$ in the (continuous) least-squares sense.

4. [**25** points] Consider the matrix

$$A = \begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix}.$$

(a) [**5** points] Find all real eigenvalues of A, with their algebraic multiplicities.

(b) [**5** points] For each eigenvalue of A, find a basis of the associated eigenspace. What are the geometric multiplicities of the eigenvalues of A?

(c) [**5 points**] Does there exist an eigenbasis for the matrix A? Motivate your answer

(d) [**5 points**] Is A diagonalizable? Motivate your answer.

(e) [**5 points**] Determine the eigenvalues of A^2, with their algebraic and geometric multiplicities.