ETHICS PLEDGE: I agree to complete this exam without unauthorized assistance from any person or person's work, materials or device.

Your name (print): ________________________ Section:________

Signature: ________________________________ Date: ________

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110.201 Prof. Zucker Exam 2: Nov. 20, 2006 Time: 85 minutes

Name: **Section Number:**

No books, no notes, no calculators or other electronic devices. Write legibly, and show all relevant work—or risk losing credit. Answer what is asked, and only what is asked.

[5,10] 1. This problem is about the linear system:

$$x_1 + x_2 = 1, \quad x_1 - x_2 = 0, \quad 2x_1 + x_2 = 0.$$

a) Determine that there are no solutions to this system.

b) The least-squares (approximate) solution of the system is the exact solution for a system with the same left-hand sides, but different constants on the right-hand sides. Determine both those constants and the least-squares solution.

[10] 2. Are the linear spaces P_3 and $\mathbb{R}^{2\times 2}$ isomorphic? **Explain.**

[5,10] 3. a) Determine all unit vectors that are perpendicular to the plane in $\mathbb{R}^3$ (whose equation is) $x_1 + 2x_2 - 3x_3 = 0$.

b) Determine an orthonormal basis $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ of $\mathbb{R}^3$ for which $\{\vec{u}_1, \vec{u}_2\}$ is a basis for the plane $x_1 + 2x_2 - 3x_3 = 0$.

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[10] 4. This problem is about functions $T : V \to W$, when V and W are linear spaces.

a) What is meant when one says "T is linear"? That is, give the definition of *linear*.

b) Let $V = \mathbb{R}^{2\times 2}$, $W = \mathbb{R}$. Explain in terms of a) why the function $T(A) = \det(A)$ is not linear.

[15] 5. Let A be the 4×4 matrix $\begin{bmatrix} 0 & 3 & 6 & 4 \\ 0 & 4 & 6 & 3 \\ 0 & 0 & 0 & 2 \\ 2 & 5 & 5 & 1 \end{bmatrix}$.

a) Calculate the determinant of A.

b) Determine whether A^5 is invertible.

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[20] 6. We define a linear transformation $\Phi : P_3 \to \mathbb{R}^2$ by $\Phi(f) = f(0)\vec{e}_1 + f(1)\vec{e}_2$.

a) Show that Φ is, in fact, linear.

b) Define the *rank* of a linear transformation T. Determine directly that the rank of Φ is 2.

c) Define the *nullity* of a linear transformation T.

d) Complete this statement of the rank-nullity formula, and use it to determine the nullity of Φ.

Let $T : V \to W$ be a linear transformation of finite-dimensional linear spaces. Then ...

[15] 7. In $\mathbb{R}^4$, let $\vec{v}_1 = \vec{e}_1$, $\quad \vec{v}_2 = 9\vec{e}_1 + 10\vec{e}_2$, $\quad \vec{v}_3 = 13\vec{e}_1 - 12\vec{e}_2 + \vec{e}_3$, $\quad \vec{v}_4 = 11\vec{e}_1 + 8\vec{e}_2 - 7\vec{e}_3 + 14\vec{e}_4$. Determine whether $\mathfrak{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is a basis of $\mathbb{R}^4$.

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[10] 8. An inner product on continuous functions on $[0, 2\pi]$ is defined by

$$< f, g > \; = \int_0^{2\pi} f(t)\, g(t)\, dt.$$

a) Show that the functions $\sin t$ and $\cos t$ on $[0, 2\pi]$ are orthogonal.

b) Show that with respect to the inner product on continuous functions on $[0, 1]$, namely

$$< f, g > \; = \int_0^{1} f(t)\, g(t)\, dt,$$

$\sin t$ and $\cos t$ on $[0, 1]$ are *not* orthogonal.