Johns Hopkins University
Math 201, Spring 2007
Name:
Section:

Midterm Exam # 2

Time: 50 minutes

No books, notes, calculators. Please explain carefully all steps leading to your solutions, or risk losing credit.

Problem 1: (6 points=2+1+1+2) Consider the plane E in $\mathbb{R}^3$ with equation $x_1 + 2x_2 + x_3 = 0$, and let p denote the orthogonal projection onto E.

1. If (u_1, u_2) is an orthonormal basis of E, write the formula for $p(v)$ in terms of u_1 and u_2 (where v is any vector in $\mathbb{R}^3$).
2. Find a basis of E.
3. Find an orthonormal basis of E.
4. Find the matrix for p in the standard basis of $\mathbb{R}^3$.

Problem 2: (9 points=3+2+1+1+2) Consider the linear space P of polynomials with real coefficients.

1. Are the following subsets of P linear subspaces? Explain why.

   -the set E_0 of polynomials p such that $p(0) = 0$

   -the set E_1 of polynomials p such that $p(1) = 1$

   -the set P_2 of polynomials of degree 2 or less

   Consider the linear map f from P_2 to P_2 defined by $f(p(x)) = p''(x) + 3p'(x)$.

2. Find the kernel of f. Is f an isomorphism?

3. Find the matrix for f in the standard basis $(1, x, x^2)$ of P_2.

4. Prove that the vectors $p_1 = 2 + x$, $p_2 = 3$, $p_3 = 1 + 2x + 3x^2$ are linearly independant.

5. Find the matrix for f in the basis (p_1, p_2, p_3).

Problem 3: (5 points=1+2+2)
Consider the matrix:

$$M = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix}$$

and let v_1,v_2,v_3 denote its column vectors.

1. Prove that v_1,v_2,v_3 are linearly independant.
2. Perform the Gram-Schmidt process on (v_1, v_2, v_3).
3. Write the QR-factorization of M.