April 1, 2009

Name ....
Section/ Name of your TA ....

MIDTERM EXAM 2 *100pts.*
MATH 201 VER ****

- **There are 6 pages in the exam including this page.**
- **Write all your answers clearly. You have to show work to get points for your answers.**
- **Read all the questions carefully and make sure you answer all the parts.**
- **You can write on both sides of the paper. Indicate that the answer follows on the back of the page.**
- **Use of Calculators is *not* allowed during the exam.**

(1) ..../22

(2) ..../20

(3) ..../22

(4) ..../36

Total ....../100

(1) *22 pts.* Let $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 0 & 0 \\ 4 & 1 & 0 \end{bmatrix}$.

(a) Find the determinant of A. Show work

(b) Find the classical adjoint of A. Show work

(c) What is the inverse of A? Show work.

(2) *20 pts.* Let $\mathcal{P}_1$ be the set of polynomials with degree less than or equal to 1, that is, $\mathcal{P}_1 = \{f(t) = a_0 + a_1 t : a_0, a_1 \in \mathbb{R}\}$. Then both $\mathcal{B}_1 = \{1, t\}$ and $\mathcal{B}_2\{1, t-1\}$ are bases of $\mathcal{P}_1$.

(a) What is the S matrix that transforms a vector in $\mathcal{B}_2$-coordinates into $\mathcal{B}_1$-coordinates. Show work

(b) Let $T : \mathcal{P}_1 \to \mathcal{P}_1$ be the transformation defined as $T(a_0 + a_1 t) = a_0 + a_1(2t - 1)$. Find the matrix B of the transformation T with respect to the basis $\mathcal{B}_2$. Show work.

(c) Let $f \in \mathcal{P}_1$ be written as $[f]_{\mathcal{B}_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in $\mathcal{B}_2$-coordinates. Let T be as in part (b), then find $[T(f)]_{\mathcal{B}_1}$, that is, find $T(f)$ in $\mathcal{B}_1$-coordinates. Show work.

(3) *22pts.* (a) Let $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ -1 \end{bmatrix} \right\}$ be two different sets in $\mathbb{R}^4$ spanning the same subspace. Which of these two sets are orthogonal? Show work.

(b) Let $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ be a basis of a subspace V of $\mathbb{R}^4$. Find an orthonormal basis of V. Show work.

(4) *36 pts.* These are all short answer questions. Explain your answer. Each of these problems is worth 12 points.

(a) Let $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 6$. Find $\det\begin{bmatrix} a-d & b-e & c-f \\ g & h & i \\ 4d & 4e & 4f \end{bmatrix}$. Explain your answer.

(b) The space of polynomials of degree less than or equal to 1, $\mathcal{P}_1$ is isomorphic to the space of complex numbers $\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$. State true or false. Give reasons.

(c) Find the volume of the parallelepiped which has the vectors $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ as three edges. Show work and explain your answer.