Math 201 Midterm II Spring 11

1. (a) $T : P_2 \to P_2$ be the linear transformation defined by $T(f) = f + f''$. Let $\mathcal{S} = (1, x, x^2)$ be the standard basis for P_2. Find the $\mathcal{S}$-matrix A for T.

(b) Let $\mathcal{B} = (1 + x, x + x^2, 1 + x^2)$ be another basis for P_2. Let B be the $\mathcal{B}$-matrix for the linear transformation T. Find the invertible matrix S such that $B = S^{-1}AS$.

2. True or False. Justify your answer.

(a) There exists an invertible 2×2 matrix S such that $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = S^{-1} \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} S$.

(b) If $\mathbf{v}_1, \mathbf{v}_2$ is a basis for $\mathbf{R}^2$. then $T(\mathbf{v}_1), T(\mathbf{v}_1)$ is a basis for $\mathbf{R}^2$ for any linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$.

3. Find an orthonormal basis for $\text{Ker}(\text{Proj}_V)$ where $\text{Proj}_V : \mathbf{R}^4 \to \mathbf{R}^4$ is the orthogonal projection onto the subspace $V = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$, where $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$.

4. True or False. Justify your answer.

(a) If A and S are orthogonal matrices, then $S^{-1}AS$ is orthogonal as well.

(b) Let A and B be two 2×2 matrices. If BA is orthogonal then A and B are orthogonal.

5. Find the least squares solution to the system $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Find the orthogonal projection of $\mathbf{b}$ onto the subspace $\text{Im}\, A$.