1. Let $\phi$ be a function such that $\frac{\partial \phi}{\partial x}(0,0,0) = 2$, $\frac{\partial \phi}{\partial y}(0,0,0) = 3$ and $\frac{\partial \phi}{\partial z}(0,0,0) = 4$.

(a) Let $w(t) = \phi(c(t))$, where $c(t) = t\mathbf{i} + t^2\mathbf{j} + 3t\mathbf{k}$ is a curve. Find $\frac{dw}{dt}(0)$!

(b) In which direction is the rate of increase of $\phi$ largest at the point $(0,0,0)$?

(c) Let $\mathbf{F} = \text{grad } \phi$. Find $\text{curl } \mathbf{F}$.

2. Let $F(x, y, z) = 8x^2 + \sin^2(\pi x) + y^2 + 2z^2$ and consider the surface $S$ given by $F(x, y, z) = 12$ and point $p = (1, 2, 0)$ on the surface.

(a) Find the equation for the tangent plane at $p$.

(b) Show that it is possible to solve for $x$ on $S$ as a function of $(y, z)$ close to $p$.

(c) Let $x = f(y, z)$ be the function in (b). Find $Df(2, 0)$.

3. Let $\mathbf{G} = -yi + xj$ be a vector field.

(a) Sketch the vector field $\mathbf{G}$ at the points $(1, 0)$, $(0, 1)$, $(-1, 0)$ and $(0, -1)$ and sketch the flow line passing through $(1, 0)$.

(b) Find the flow lines analytically.

4. Let $\mathbf{F}(x, y, z) = (y^2 + x)\mathbf{i} - (x^2 - y)\mathbf{j} + zk$.

(a) Find $\text{curl } \mathbf{F}$.

(b) Find $\text{div } \mathbf{F}$.

(c) Find the derivative matrix $DF$ (i.e. the matrix of partial derivatives).

5. Let $f(x,y) = x \cos(x+y)$

(a) Calculate the second order Taylor polynomial of $f$ about the point $(1, -1)$.

(b) Use your answer to (a) to write down an estimate for $f(1.1, -0.8)$.

(c) Use the linear approximation to find an estimate for $f(1.1, -0.8)$. 