1. (a) Use Lagrange’s method to find the point on the plane $x + 2y + 3z = 1$ closest to the point $(2, 1, 0)$.

(b) Find the critical points of $f(x, y) = x^2 + y^2 + x + y + 1$ and determine if they are local max min or saddle points.

2. (a) Let $\mathbf{F} = \mathbf{i} + \mathbf{j}$ and let $\mathbf{c}_1$ be the curve $\mathbf{c}_1 = \cos t \mathbf{i} + 2 \sin t \mathbf{j} + t \mathbf{k}$, $0 \leq t \leq \pi/2$.

Find the line integral $\int_{\mathbf{c}_1} \mathbf{F} \cdot ds$.

(b) Let $\mathbf{F} = \mathbf{i} + \mathbf{j}$ and let $\mathbf{c}_2$ be the straight line segment from $(1, 0, 0)$ to $(0, 2, \pi/2)$.

Find the line integral $\int_{\mathbf{c}_2} \mathbf{F} \cdot ds$.

3. Find $\iint_R (x^2 + y^2)xy \, dxdy$, where $R$ is the region

$R = \{(x, y); 1 \leq y^2 - x^2 \leq 2, \ 2 \leq xy \leq 3\}$

by making the change of variables $u = y^2 - x^2$ and $v = xy$.

4. Let $R$ be the 3-dimensional region $R = \{x^2/4 + y + z^2/4 \leq 1, y \geq 0\}$. Let $S$ be the surface of $R$ with the normal oriented outwards. Note that $S$ has two parts $\{x^2/4 + y + z^2/4 = 1, y \geq 0\}$ and $\{y = 0, x^2/4 + z^2/4 \leq 1\}$.

(a) Find the area of $S$.

(b) Find the flux of $\mathbf{F} = x\mathbf{i} - y\mathbf{j} + \mathbf{k}$ through $S$; $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$.

5. A parametrization of the solid torus $E$ is given

$$x = 2 \cos \theta + r \cos \theta \cos \phi, \quad y = 2 \sin \theta + r \sin \theta \cos \phi, \quad z = r \sin \phi,$$

where $0 \leq r \leq 1$, $0 \leq \theta \leq 2\pi$ and $0 \leq \phi \leq 2\pi$. A parametrization for the surface of the torus $T$ is obtained from this by putting $r = 1$.

(a) Calculate the surface area element $dA$ in terms of $d\theta d\phi$ and use it to calculate the area of the surface of the torus $T$.

(b) Calculate the volume element $dV$ in terms of $dr d\theta d\phi$ and use it to calculate the volume of the solid torus $E$. 