The second midterm exam will take place on Monday, 15 April 2019, during class. It will be 50 minutes long, starting **promptly** at 12:00 pm (noon), and at 1:30 pm, respectively.

You will **not** be allowed to use any textbooks, notes, calculators, cellphones or other devices in the exam. You will have to show all your working and explain your answers clearly to obtain full credit. You will be provided with a list of several possibly helpful formulas, this list is included at the end of this file.

The exam will be about the material covered in class from the sections 3.3–3.6, 4.1–4.3, 7.1–7.8, 9.1. You should be comfortable with all the examples from class and the homework problems, including the ninth set of homework problems that is due April 12th.

If you have any questions or concerns, come to see me after class or in my office hours or send me an email.

Here are several practice problems that I recommend you to do as part of your preparation for the second midterm exam.

**Problem 1.** Find the general solution to the differential equation

\[ y'' - 2y' + y = 4e^t \ln(t), \quad t > 0. \]

**Problem 2.** Find the general solution to the differential equation

\[ y'' - 4y' + 4y = te^{2t}. \]

**Problem 3.** Consider the differential equation

\[ 3t^2y'' + 9ty' - 9y = 0, \quad t > 0. \]

Given that \( y_1(t) = t^{-3} \) is a solution, find a fundamental set of solutions.

**Problem 4.** Consider the system of two linear ODEs

\[
\begin{align*}
x_1' & = -2x_1 + x_2, \\
x_2' & = -2x_2
\end{align*}
\]

with initial values \( x_1(0) = 2 \) and \( x_2(0) = 3 \). Determine the solution.

**Problem 5.** Determine the general vector solution to the following system of two linear ODEs

\[
\begin{align*}
x_1' & = 3x_1 + 5x_2, \\
x_2' & = -2x_1 - 3x_2
\end{align*}
\]

**Problem 6.** Consider the system of two linear ODEs

\[
\begin{align*}
x_1' & = x_1 + 3x_2 \\
x_2' & = x_1 - x_2
\end{align*}
\]

(a) Determine the general vector solution to this system.

(b) Draw the phase portrait for this system.
Some possibly helpful formulae.

- $W(y_1, y_2) = y_1 y_2' - y_1' y_2$
- $y'' + p(t)y' + q(t)y = 0$, $y_2(t) = v(t)y_1(t)$, $y_1 y'' + (2y_1' + py_1) y' = 0$
- $y'' + p(t)y' + q(t)y = g(t)$, $y(t) = c_1 y_1(t) + c_2 y_2(t) + Y(t)$
- $Y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$, $u_1'(t) = -\frac{g_2(t) - g(t)}{W(y_1, y_2)(t)}$, $u_2'(t) = \frac{y_1(t) g(t)}{W(y_1, y_2)(t)}$
- $\tilde{y}(t) = c_1 e^{\lambda t}(\tilde{a} \cos(\mu t) - \tilde{b} \sin(\mu t)) + c_2 e^{\lambda t}(\tilde{a} \sin(\mu t) + \tilde{b} \cos(\mu t))$
- $\tilde{x}(t) = c_1 \tilde{\xi} e^{rt} + c_2 (\tilde{\xi} e^{rt} + \tilde{\eta} e^{rt})$
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Good luck and all the best for the exam!
Jonas