Johns Hopkins Mathematics Tournament

April 23, 2005

Geometry Question Paper

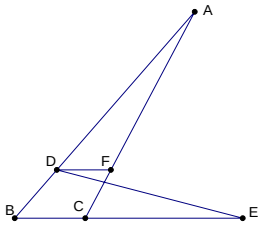
1. A circle with diameter 23 is cut by a chord AC. Two different circles can be inscribed between the large circle and AC. Find the sum of the two radii.

2. Regular hexagon $ABCDEF$ is inscribed in rectangle $PQRS$ with $AB = 1$, A and B on side PQ, C on side QR, D and E on side RS, and F on side SP. What is the area of $PQRS$?

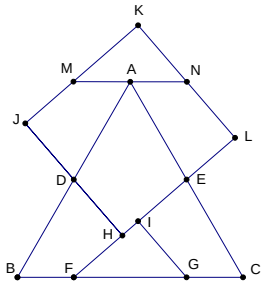
3. Isosceles triangle ABC has angle $BAC = 135^o$ and $AB = 2$. What is its area?

4. Given an isosceles trapezoid $ABCD$ with $AB = 6$, $CD = 12$, and area 36, find BC.

5. Equilateral triangle ABC has $AD = DB = FG = AE = EC = 4$ and $BF = GC = 2$. From D and G are drawn perpendiculars to EF intersecting at H and I, respectively. The three polygons $ECGI$, FGI, and $BFHD$ are rearranged to $EANL$, MNK, and $AMJD$ so that the rectangle $HLKJ$ is formed. Find its area.



Problem 5



Problem 6

6. Line DE cuts through triangle ABC, with DF parallel to BE. Given that $BD = DF = 10$ and $AD = BE = 25$, find BC.

7. Equilateral triangle ABC is inscribed in a circle with radius 6. Find the area of the region enclosed by AB, AC, and the minor arc BC.

8. The square $DEAF$ is constructed inside the 30^o–60^o–90^o triangle ABC, with the hypotenuse $BC = 4$, D on side BC, E on side AC, and F on side AB. What is the side length of the square?

9. A square with side length 1 is inscribed in a hemisphere such that one side of the square is on the hemisphere's diameter. What is the semicircle's perimeter?

10. Rectangle $ABCD$ has sides in the ratio of $\sqrt{2}$ to 1. If DEC is an isosceles right triangle, with E inside the rectangle, find angle AEB.