Math 421: Dynamical Systems & Chaos – Professor Haskins
Office hours for the class are: M 3-4, Tu 9-10, Th 11-12.

Homework 5: 10 Oct 00
Due: 17 Oct 00 at start of class

Remember to show all your working and to write clearly, neatly.

Part 0: Midterm Prep

Remember that you have a midterm on Tuesday, October 17 and no class on Monday, October 16 (Fall Break). Be sure to do some revision for this midterm. The midterm will have 2 parts: an in-class exam and a take-home exam. The take-home exam will be handed out during class on Tuesday and due in class on Wednesday. It will include some work with Matlab.

Part I: Discrete time systems

From the photocopy of Devaney’s book “A First Course in Chaotic Dynamical Systems”:
• Read Chapter 7
• Questions 1-8 from Chapter 7 Exercises, p80.

Part II: Matlab work

1. Adapt your Matlab scripts from the previous homeworks to carry out Computer Experiment 6.4 from Devaney Chapter 6, p65. You may do the same for the logistic function too if you wish (as suggested in the Notes section of the question) but you should not hand this part in.

2. We have seen that for \( c > 1/4 \) all orbits of \( Q_c = x^2 + c \) eventually tend to \( \infty \). In particular the orbit of 0 ends up going to \( \infty \). For \( c = 1/4 \), \( Q_c \) has exactly 1 fixed point at \( x = 1/2 \) which is semi-stable. In particular, the orbit of 0 tends towards the fixed point 1/2.

   Now look at the orbit of 0 under \( Q_c \) for \( c = (1/4) + \epsilon \), where \( \epsilon \) is some small positive number. We know that for any \( \epsilon > 0 \) the orbit of 0 must tend to \( \infty \), whereas for \( \epsilon = 0 \) it gets “stuck” at the fixed point \( x = 1/2 \). In other words as \( \epsilon \) tends to 0 the orbit of 0 spends more and more time close to the fixed point \( x = 1/2 \).

   Investigate how much time the orbit of 0 spends near the fixed point in the following way. For \( \epsilon = 1/2, 1/5, 1/10, 1/50, 1/100, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6} \) record in a table the following information:
• $\epsilon$
• the number of iterations, $N(\epsilon)$, needed before $x_n > 2$ (with $x_0 = 0$)
• the product $N(\epsilon) \times \epsilon$
• the product $N(\epsilon) \times \sqrt{\epsilon}$

What do you notice about $\sqrt{\epsilon} \times N(\epsilon)$?

Part IV: Project

Start looking at project information. You must decide on a project by Tuesday October 24 (when you will hand in HW 6) and hand in a brief description of what you intend to do for your project that day. See separate sheet “Project Info” (to be handed out) for more information.