SOLUTIONS TO PROBABILITY QUESTION FROM LAST TIME

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Exercise 1. The letters of the alphabet are written on 26 cards. Two cards are chosen at random. What is the probability that at least one of them is a consonant?

There are different ways one can solve this; I’ll write 3 possible solutions. Compare with what you’ve done. On an exam, you’d have to write only one correct solution, of course.

Solution 1.

Let

\[ E = \text{event that at least one is a consonant}. \]

Let’s compute

\[ E^c = \text{event that neither is a consonant} \]

instead and then use

\[ P(E) = 1 - P(E^c). \]

Let A be event that the first card is a consonant and B be event that the second card is a consonant. The event \( E^c \) is the event that the first card is not a consonant and the second card is not a consonant, so

\[ E^c = A^c \cap B^c. \]

Note that \( A^c \) and \( B^c \) are dependent, and \( P(A^c) = \frac{5}{26} \) and \( P(B^c | A^c) = \frac{4}{25} \). Thus

\[ P(A^c \cap B^c) = \frac{5}{26} \times \frac{4}{25} = \frac{2}{65}. \]

Thus

\[ P(E) = 1 - \frac{2}{65} = \frac{63}{65}. \]

Note 1. We were after the probability of \( E = A \cup B \). Instead, we computed the probability of \( E^c = (A \cup B)^c \) and what we really did is use DeMorgan’s law to write \((A \cup B)^c = A^c \cap B^c\).

Solution 2.

Let

\[ E = \text{event that at least one is a consonant}. \]

Let A be event that the first card is a consonant and B be event that the second card is a consonant. so

\[ E = A \cup B. \]

Now A and B are not independent, so recall that

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B). \]
Now the unconditional probability of A or B is \( P(A) = P(B) = \frac{21}{26} \). Also, we can compute
\[ P(A \cap B) = P(A)P(A \mid B) = \frac{21}{26} \times \frac{20}{25}. \]

Therefore,
\[ P(E) = \frac{21}{26} + \frac{21}{26} - \frac{21}{26} \times \frac{20}{25} = \frac{42}{26} - \frac{21 \cdot 20}{26 \cdot 25} = \frac{63}{65}. \]
(This was not hard at all to simplify)

**Solution 3.**

Let
\[ E = \text{event that at least one is a consonant}. \]

Let’s compute
\[ E^c = \text{event that neither is a consonant} \]

instead and then use
\[ P(E) = 1 - P(E^c). \]

Now let’s compute \( P(E^c) \) in a different way. \( E^c \) is the event that neither card is a consonant, or equivalently, that both cards are vowels. There are \( \binom{26}{2} \) ways to choose 2 cards out of 26, and there are \( \binom{5}{2} \) ways to choose 2 vowels out of 5. Therefore,
\[ P(E^c) = \frac{\binom{5}{2}}{\binom{26}{2}} = \frac{\frac{5!}{2! \cdot 3!}}{\frac{26!}{2! \cdot 24!}} = \frac{5! \cdot 24!}{2! \cdot 3! \cdot 26!} = \frac{2}{65}. \]
(This was not hard at all to simplify)

Therefore,
\[ P(E) = 1 - \frac{2}{65} = \frac{63}{65}. \]