0.* Let $X$ be a path-connected space, and $x_0 \in X$. Describe
(i) the fundamental group of the cylinder of $X$
   \[ \text{Cyl}(X) := X \times I; \]
(ii) the fundamental group of the cone of $X$, defined as the quotient
   \[ \text{Cone}(X) := \text{Cyl}(X)/\sim, \]
   where $\sim$ is the equivalence relation generated by $(x, 1) \sim (y, 1)$ for any $x, y \in X$.
(iii) the fundamental group of the suspension of $X$, defined as the quotient
    \[ \Sigma X := \text{Cyl}(X)/\sim, \]
    where $\sim$ is the equivalence relation generated by $(x, 1) \sim (y, 1)$ and $(x, 0) \sim (y, 0)$  
    for any $x, y \in X$.

1. A space is said to be simply connected if it is path-connected and its fundamental group
   is the trivial group. Say whether each of the following statements is true or false and justify.
   (i) Any subspace of a simply connected space is simply connected.
   (ii) Any quotient of a simply connected space is simply connected.
   (iii) The product of two simply connected spaces is simply connected.
   (iv) Every contractible space is simply connected.
   (v) Every simply connected space is contractible.

2. Find a space whose fundamental group is
   (i) $\mathbb{Z}/2 \times \mathbb{Z}$.
   (ii) $\mathbb{Z}/2 * \mathbb{Z}$.
   (iii) $(\mathbb{Z} \times \mathbb{Z}) * \mathbb{Z}$.
   (iv) $(\mathbb{Z} * \mathbb{Z}) \times \mathbb{Z}$.
   (v) $\mathbb{Z} * \cdots * \mathbb{Z}$.
      $n$ copies