0.* Indicate the option you prefer.

(i) Have two midterms (which will be in class on March 15 and May 3) and no final, with grade computed as 60% problem sets, 40% midterms.

(ii) Have two midterms (which will be in class on March 15 and May 3) and a final (which will be on May 17), with grade computed as 50% problem sets, 25% midterms, 25% final.

1. Consider the topologies $T_{disc}, T_{codisc}, T_{fin}, T_{cont}, T_{st}, T_{K}, T_{up}, T_{uplim}$ on $\mathbb{R}$. For each topology,

(i) say if $K := \{ \frac{1}{n} \mid n \in \mathbb{N} \}$ is closed.

(ii) say if $K \cup \{0\}$ is closed.

(iii) say if $\mathbb{R}$ is Hausdorff.

(iv) say if $\mathbb{R}$ is connected, except for the topology $T_{K}$. You can assume that $(\mathbb{R}, T_{st})$ is connected, which will be proven in class.

(v) say if $\mathbb{Q}$, endowed with the subspace topology, is connected.

Justify each answer. To make the exercise shorter, it is useful to keep in mind that these topologies are related.

2. Prove that a space $(X, T)$ is Hausdorff if and only if the diagonal $\Delta := \{(x, x) \mid x \in X\}$ is closed in $(X \times X, T \ast T)$