MATH 111 – SECOND MIDTERM EXAM

(1) State the Archimedean property of the real numbers.

(2) If $f : (a, b) \to \mathbb{R}$ and $x \in (a, b)$, state in terms of ε-δ what it means for f to be continuous as x.

(3) How many solutions are there to the equation: $x^3 - 3x + 1 = 0$? Justify your answer fully. (HINT: plug in a few simple numbers to get an idea of the graph.)

(4) Let A, B be two sets of real numbers with $x \leq y$ for all $x \in A$ and $y \in B$. Show that $\sup A \leq \inf B$.

(5) *Do one of the following two problems. If you do both, specify which one you want graded.*

(a) If $f(x)$ is continuous on $[-1, 1]$ and differentiable at $x = 0$, and $f(0) = 0$, show that there is a constant C so that: $|f(x)| \leq C|x|$ for all $x \in [-1, 1]$. (HINT: first show that the function $g(x) = f(x)/x$, has a continuous extension to $[-1, 1]$)

(b) Recall that we say $\lim_{x\to\infty} f(x) = L$, for some real number L, if for any $\varepsilon > 0$ there is a number M such that $|f(x) - L| < \varepsilon$ if $x \geq M$. Show that if $f : [0, \infty) \to \mathbb{R}$ is continuous and $\lim_{x\to\infty} f(x) = L$, then f is uniformly continuous on $[0, \infty)$. (HINT: consider the intervals $[0, M]$ and $[M, \infty)$ separately)

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