(1) State the Cayley-Hamilton Theorem. Solution. If $T : V \rightarrow V$ is a linear transformation of a finite dimensional vector, and $p(x)$ is the characteristic polynomial of $T$, then $p(T) = 0$.

(2) Answer the following True or False. Be sure to justify your response either with an explanation or with an example. Here, $T$ is a linear transformation from an $n$-dimensional vector space to itself.

(a) $T$ is diagonalizable if and only if the minimal polynomial is a product of distinct linear factors. True.

(b) Eigenvectors with distinct eigenvalues are linearly independent. True.

(c) $T$ is nilpotent if and only if the characteristic polynomial of $T$ is $x^n$. True.

(3) Compute the characteristic and minimal polynomials of the matrix:
$$
\begin{pmatrix}
1 & 0 & 2 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 2 & 1 \\
0 & 0 & 0 & 0 & 2
\end{pmatrix}
$$

Solution. The characteristic polynomial is $(x - 1)^3(x - 2)^2$, whereas the minimal polynomial is: $(x - 1)^2(x - 2)^2$. 