Homework II
Math 113

This homework is due on Sept 22, 2008. Read Appendix 2,3 for chapter 4.

ex.1. a. Write down whether the following sets are bounded above. If so what is its least upper bound?
   b. Write down whether the following sets are bounded below. If so what is its greatest lower bound?
   
   (i) \( \{ x \in \mathbb{R} : (x^2 - 1) < 0 \} \)
   
   (ii) \( \{ x \in \mathbb{R} : |x| > 3 \} \)
   
   (iii) \( \{ \frac{1}{x} : x > 1, x \in \mathbb{R} \} \)
   
   (iv) \( \{ \frac{1}{n} + (-1)^n : n \in \mathbb{N} \} \)

ex.2. Let \( A = \{ p \in \mathbb{Q} : 0 < p^2 < 2 \} \) and \( B = \{ p \in \mathbb{Q} : 2 < p^2 \} \).
   
   (i) Let \( q = p - \frac{p^2 - 2}{p + 2} \). Show that \( q > p \) if \( p \in A \) and \( q < p \) if \( p \in B \).
   
   (ii) Show that \( q^2 - 2 = \frac{2(p^2 - 2)}{(p+2)^2} \).
   
   (iii) Show that \( A \) does not have least upper bound in \( \mathbb{Q} \) and \( B \) does not have greatest lower bound in \( \mathbb{Q} \). (Hint: Use (i) and (ii) to show that if \( q \) is in \( A \) then \( q \) cannot be an upper bound of \( A \).)

ex.3. Use statements proved in class about natural numbers to show that: “If \( x, y \in \mathbb{R} \) and \( x > 0 \) then there is a positive integer \( n \) such that \( nx > y \).”

   (Hint: Consider all the different case for \( y = 0, y > 0 \) and \( y < 0 \)).

ex 4 Write down the interval on which the following functions are one to one and then compute their inverse over that domain.

   (i) \( f(x) = x^3 \).
   
   (ii) \( f(x) = \frac{1}{x^2-1}, x \neq 1 \)

ex 5 Shade the region described by the following. Some may be written in terms of polar coordinates.

   (i) \( 2 < r < 5 \)
   
   (ii) \( x^2 < y < 5x^2 \)
   
   (iii) \( |x - y| < 1 \)

\[1\text{This exercise shows that \( \mathbb{Q} \) does not have the least upper bound property}\]
ex 6 Complete squares if necessary and write down which conic section the following equation represents. Make sure you write the final equation down. Please show your work.

(i) \( x^2 + y^2 = x \)
(ii) \( 2x^2 + 4x = y^2 + 6 \)
(iii) \( r = \frac{1}{2 + \cos \theta} \)
Extra Practice Problems
Math 113

(I) 

a. Let $A$ be a subset of real numbers which is bounded below. Let $-A$ be the set \{-x : x \in A\}. Then show that $-A$ is bounded above.

b. A set $S$ has the greatest lower bound property if whenever a subset $A$ of $S$ is bounded below (that is, has a lower bound) then $A$ has the greatest lower bound in $S$. Use the previous exercise to show that the set of real numbers have the greatest lower bound property. (Hint: The set of real numbers has the L.U.B property!)

(II) Ex.3,10 from Chapter 2 of the textbook.

(III) Solve 5 from Chapter 8 of the textbook.

(IV) Ex 5 (i) from Chapter 4 Appendix 3.

(V) Solve 1,3, 14 from Chapter 3.

(VI) Ex 4(ii),(vi),(viii) from Chapter 4.