Here are some practice problems from Chapters 19, 20, 22, 23, and 24

ex.1 Show that the series
\[ \sum_{n=0}^{\infty} \frac{x^n}{n!} \]
converges uniformly on \([-M, M]\) for any \(M \in \mathbb{R}\).

ex.2 Define a sequence of functions
\[ f_n(x) = x^n \]
on the interval \([0, 1]\). Let
\[ f(x) = \begin{cases} 
0 & \text{when } 0 \leq x < 1 \\
1 & \text{when } x = 1
\end{cases} \]
(i) Show that \(f_n\) converge pointwise to \(f\) but not uniformly.

(ii) Show that for any \(0 < b < 1\), the functions \(f_n\) converge to \(f\) uniformly on \([0, b]\).

ex.3 Solve the following problems from chapter 19.

(i) Problems 3(vi), 4(x), 4(vi), 6(viii) (same for 3rd edition)


(iii) Problem 22(a) (Problem 21(a) 3rd edition).

ex.3 Compute the Taylor polynomial for \(f(x) = \cos x\) around 0 of degree \(n\) for any \(n \in \mathbb{N}\). Show that this polynomial converges uniformly at \(n\) tends to \(\infty\).

ex.3 Solve the following problems from chapter 22. (both editions)

(i) Problem 1(vi) , (viii)

(ii) Problem 3

(iii) Problem 9(iv)

(iv) Problem 13.