9 February, 2010

MATH 212 Homework 2
Due on February 19, 2010

(1) Check if the following subsets $W$ of the given vectorspace $V$ are subspaces of $V$.

(a) \[ W = \{(a, b, c) \in \mathbb{R}^3 \mid a + 3c + b = 0\} \subset \mathbb{R}^3 = V. \]

(b) \[ W = \{f : \mathbb{F}_3 \to \mathbb{F}_3, f(x) = a + ax + a^2 x^2 \mid a \in \mathbb{F}_3\} \subset \mathcal{P}_2(\mathbb{F}_3) = V. \]

(c) \[ W = \begin{bmatrix} s & 3t \\ s + t & 0 \end{bmatrix} \mid s, t \in \mathbb{C} \subset \mathcal{M}_{2 \times 2}(\mathbb{C}) = V. \]

(d) Let $F$ be a field.
\[ W = \{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in F, A^2 = A\} \subset \mathcal{M}_{2 \times 2}(F) = V. \]

(2) Show that the set of all even functions $V_e$,
\[ V_e = \{f : \mathbb{R} \to \mathbb{R} \mid f(-x) = f(x)\}. \]

is a vectorspace over $\mathbb{R}$ with addition and scalar multiplication as defined on $\mathcal{F}(\mathbb{R}, \mathbb{R})$, the set of functions $\mathbb{R} \to \mathbb{R}$.

(3) Page 22, Exercise 23. (from your textbook)

(4) A proof similar to of (2) shows that the set of all odd functions
\[ V_o = \{f : \mathbb{R} \to \mathbb{R} \mid f(-x) = -f(x)\}. \]

is a subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$.

Prove that $V_o + V_e = \mathcal{F}(\mathbb{R}, \mathbb{R})$ and $V_o \cap V_e = \{0\}$

(5) Page 22, Exercise 27. (from your textbook)