Assignment #1, due Thursday, September 12, 2002:


Problems:

To receive full credit on a problem, your solution must be written carefully and completely, using sentences and formulas as appropriate.


Part II.

1. Let

\[ f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \]

Verify the following statements, in the order given:

a) \( f \) is continuous at 0.

b) \( f'(0) \) exists and equals 0.

c) \( f'' \) is continuous at 0.

d) \( f'''(0) \) exists and equals 0.

e) \textbf{Extra credit.} Prove by induction that \( f^{(n)}(0) = 0 \) for all positive integers \( n \). What is the Taylor series for \( f \) at 0?

2. Determine whether each of the following series converges and say why.

a) \[ \sum_{n=1}^{\infty} \frac{1}{n} \]

b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \]

c) \[ \sum_{n=1}^{\infty} \frac{\sigma_n}{n} \] where \( \sigma_n = \begin{cases} -1 & \text{if } n \text{ is divisible by 3} \\ 1 & \text{otherwise} \end{cases} \]