Assignment Due September 19, 2002:

Reading: 12.6–12.12.

Problems:

Part I: 12.4: 4, 5, 12
12.8: 5, 6, 7, 19, 20, 25

Part II. Least Squares Fit to Data:

Suppose a laboratory experiment yields data points \((x_1, y_1), (x_2, y_2), \ldots , (x_n, y_n)\). We wish to find the “best” approximating equation of the form \(y = mx + b\) for the data. To do this, we let \(\tilde{y}_i = mx_i + b\), for \(1 \leq i \leq n\). (The constants \(m\) and \(b\) are not yet determined.) We define:

- the data vector: \(Y = (y_1, \ldots , y_n)\)
- the approximating vector: \(\tilde{Y} = (\tilde{y}_1, \ldots , \tilde{y}_n)\)
- the error vector: \(E = Y - \tilde{Y}\)

We want to choose \(m, b\) so that \(\|E\|\) is minimal. (This is called the least squares approximation.) We let

\[ X = (x_1, \ldots , x_n), \quad A = (1, \ldots , 1) \]

so that \(\tilde{Y} = mX + bA, \ E = Y - mX - bA\). The minimality of \(\|E\|\) means that

\[ \|E\| = \|Y - mX - bA\| \leq \|Y - sX - tA\| \]

for all real numbers \(s\) and \(t\).

1. Prove that \((*)\) holds if \(E \cdot A = E \cdot X = 0\).

2. Use Problem 1 to find the best approximating equation of the form \(y = mx + b\) for the data points

\((0, -1), (1, 3), (2, 4), (3, 4)\).

3. Let \(Y\) be a vector in \(V_n\). Let \(S = \{A_1, \ldots , A_k\}\) be a set of vectors in \(V_n\), and suppose that the vector \(\tilde{Y}\) in \(L(S)\) satisfies the condition

\[ (Y - \tilde{Y}) \cdot A_j = 0 \quad \text{for} \ 1 \leq j \leq k. \]

Prove that

\[ \|Y - \tilde{Y}\| \leq \|Y - Z\| \quad \text{for all vectors} \ Z \in L(S). \]

(The vector \(\tilde{Y}\) is the best approximation to \(Y\) in \(L(S)\).)