1. Let $\lambda = dx_1 \wedge dy_1 \wedge \cdots \wedge dx_n \wedge dy_n$ be the Euclidean volume form on $\mathbb{C}^n$. Show that $(\bar{i}\partial \partial \|z\|^2)^n = c\lambda$, for a real constant $c$, and find $c$.

b) The Laplacian of a function $u$ on $\mathbb{C}^n$ is given by $\Delta u = \sum_{j=1}^n \left( \frac{\partial^2 u}{\partial x_j^2} + \frac{\partial^2 u}{\partial y_j^2} \right)$. Show that $i\partial \bar{i}\partial u \wedge (i\partial \bar{i}\partial \|z\|^2)^{n-1} = a(\Delta u)\lambda$ for a constant $a$, and find $a$.

c) A $C^2$ function $u$ on a domain $\Omega$ in $\mathbb{C}^n$ is harmonic iff $\Delta u = 0$. Prove or give a counterexample to each of the following:

i) If $f \in O(\Omega)$, then $\text{Re} f$ is harmonic.

ii) If $u$ is harmonic on $\Omega$, then $u$ is locally the real part of a holomorphic function.

2. Let $\alpha \in \mathcal{A}^p(U)$, $\beta \in \mathcal{A}^q(U)$, where $\mathcal{A}^p(U)$ denotes the space of $C^\infty$ $p$-forms on an open set $U \subset \mathbb{R}^n$. Verify:

a) $d(\alpha \wedge \beta) = (d\alpha) \wedge \beta + (-1)^p \alpha \wedge d\beta$;

b) $d^2 \alpha = 0$.

3. Let $X = (-\infty, 0] \times \mathbb{R}^{k-1}$, $B = \partial X = \{0\} \times \mathbb{R}^{k-1}$. Let $\alpha \in \mathcal{D}^{k-1}(X)$ such that $\alpha$ has compact support (i.e., there exists a compact set $K \subset X$ such that $\alpha(x) = 0$ for all $x \in X \setminus K$).

a) Write $\alpha = f dx_2 \wedge \cdots \wedge dx_k + dx_1 \wedge \beta$. Show that $\int_X dx_1 \wedge d\beta = 0$.

b) Show that Stokes' theorem holds for $X$: $\int_B \alpha = \int_X d\alpha$. 