1. (warm-up) Let $a \in \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \frac{1}{2}|x - a|$. Regard $f$ as a distribution on $\mathbb{R}$ and prove that $f'' = \delta_a$. (State and use the definition of the derivative of a distribution.)

2. Let $U$ be a bounded domain with smooth boundary in $\mathbb{R}^n$. Let $x_0 \in U$, and let $G^0(y) = G_{\mathbb{R}^n}(x_0, y)$ be the Green’s function (Newtonian potential) for $\mathbb{R}^n$. Use the derivation of the Bochner-Martinelli formula given in class to prove the analogous real formula

$$f(x_0) = \int_{\partial U} f \cdot *dG^0 - \int_U df \wedge *dG^0,$$

for $f \in C^1(\overline{U})$.

3. Let $U$ be as above. The Green’s function for $U$ is the function $G_U$ on $U \times \overline{U}$ satisfying the conditions

- $\Delta_y G_U(x, y) = \delta_x(y)$,
- $G_U(x, y) = 0$ for $y \in \partial U$.

Prove the Poisson formula:

$$f(x) = \int_U G_U(x, y) \Delta f(y) dV(y) + \int_{\partial U} f(y) \cdot *d_y G_U(x, y), \quad \text{for } f \in C^2(\overline{U}).$$

You may assume the existence of $G_U$.

Hint: Let $f_2 \in C^2(\overline{U})$ with $x \notin \text{support}(f_2)$. Show that

$$\int_{\partial U} f_2(y) \cdot *d_y G_U(x, y) = \int_U d_y G_U(x, y) \wedge *d_f(y) = \int_U G_U(x, y) d * df_2(y).$$