1. a) See Dif p. 560 Prop 3
   b) Simple subgroup criterion calc

2. See Cor 10, Dif p. 572

3. Since $k \not= E$, $3 + (4) \in E$.

   Then $\mathbb{F}(f(t)) \subset E \subset k(f(t))$. but $[k(f(t)) : \mathbb{F}(f(t))]$ is finite by previous hint, so $[k(f(t)) : E]$ must be finite.

4. Follow #7, p. 567 Dif

   Follow #8, p. 567 (or done on p. 647)

   Note $\text{Aut}(F(\xi)/F) \cong \text{PG}_2(F)$

5. If $K(\xi)$ $\&$ Galois and $\xi$ was finite, by #4, $\text{Gal}(K(\xi)/K) = \text{PG}_2(K)$ would be finite. But $[K(\xi) : K]$ infinite.

   If $\xi$ infinite, let $L$ be intermediate extension $K \subset L \subset K(\xi)$. Let $\xi \notin L$ and let $E : = k(\xi)$, and $[L(\xi) : E] < \infty$

   Since $E \subset L$, $k(\xi)$ finite on $L$. $k(\xi)$ finite on any intermediate extension (except $K$)

Consider the fixed field of $\text{Aut}(k(\xi)/k)$

   If it is $E$, extension is Galois; we are done

   So assume it is some $E \neq k$

   Then we have seen that $[k(\xi) : E] < \infty$

   And since $E$ is the fixed field, $k(\xi)$ is Galois over $E$.

   But this contradicts the fact that $\text{Aut}(k(\xi)/k) \cong \text{PG}_2(K)$ is infinite if $K$ is infinite.