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Find \( \frac{dy}{dx} \) for the curve implicitly given by \( \frac{x}{xy+1} = 2xy \)

\[
\frac{x}{xy+1} = 2xy
\]

\( \Rightarrow \frac{d}{dx} \frac{x}{xy+1} = \frac{d}{dx} (2xy) \) by doing the same thing to both sides

\[
\frac{\frac{d}{dx}x (xy + 1) - x \frac{d}{dx} (xy + 1)}{(xy + 1)^2} = \frac{d}{dx} (2xy) \text{ by quot. rule}
\]

\( \Rightarrow \frac{(xy + 1) - x \frac{d}{dx} (xy)}{(xy + 1)^2} = 2 \frac{d}{dx} (xy) \text{ by basic derivative facts} \)

\( \Rightarrow \frac{(xy + 1) - x(\frac{d}{dx}x y + x \frac{d}{dx}y)}{(xy + 1)^2} = 2 \left( \frac{d}{dx}x y + x \frac{d}{dx}y \right) \text{ by prod. rule} \)

\( \Rightarrow \frac{(xy + 1) - x \left( y + x \frac{d}{dx}y \right)}{(xy + 1)^2} = 2 \left( y + x \frac{dy}{dx} \right) \text{ by basic derivative facts} \)

\( \Rightarrow \frac{1 - x^2 \frac{dy}{dx}}{(xy + 1)^2} = 2y + 2x \frac{dy}{dx} \text{ by algebra} \)

\( \Rightarrow 1 - x^2 \frac{dy}{dx} = 2y (xy + 1)^2 + 2x (xy + 1)^2 \frac{dy}{dx} \text{ by algebra} \)

\( \Rightarrow 1 - 2y (xy + 1)^2 = (2x (xy + 1)^2 + x^2) \frac{dy}{dx} \text{ by algebra} \)

\( \Rightarrow \frac{1 - 2y (xy + 1)^2}{2x (xy + 1)^2 + x^2} = \frac{dy}{dx} \text{ by algebra} \)