The WI problem of Assignment 1: A Solution.

Recall that the problem was: We operate at the level where we don’t yet have coordinates in space, but have them in each coordinate plane, and we have fixed scaling on each of the coordinate axes. Suppose we were to project any given point in space onto the \((x, y)\)- and \((x, z)\)-planes, giving points with respective coordinates \((a, b)\) and \((c, d)\). We wish to take, say, \((a, b, d)\) as the coordinates of our point. For this to be reasonable [i.e., to treat all axes equally], we would want to know that \(a = c\). Explain carefully why this is true.

Before starting, I’d like to remind you that the projections onto coordinate planes was offered as a way to determine coordinates in space. We will see a better way to determine coordinates in answering the above question.

I also want to remind you that scribbling down your thoughts on this question will probably not constitute a WI-write up of the solution. Sure you may feel you know meant at that time, but a better test would be to show it to others, or to yourself a couple of weeks afterwards to see how long in would take you to reconstruct your thoughts from what’s written! It is very likely that you will have to edit your first attempt to write up a solution. By the same token, there is some freedom in what you might say.

**Here’s a solution.** Let \(P\) denote our point in space. It projects to a point \(P_1(a, b)\) in the \((x, y)\)-plane, and to a point \(P_2(c, d)\) in the \((x, z)\)-plane. (Note that we just have established sufficient notation to talk about the mathematical situation, and we have edited out any excess. If you need help visualizing it, use a corner of your room, with the vertical line along which the two walls intersect being the \(x\)-axis.)

The key step is: there is one and only one plane perpendicular to the \(x\)-axis that passes through the point \(P\). (This determines the \(x\)-coordinate of \(P\), as we’ll see. The same could be done for all three axes, and we thereby get the coordinates of \(P\). But that is not what we are doing now.) We assert that the points \(P_1\) and \(P_2\) both lie in this plane. Since the situation is symmetric, it is enough to consider one of them, say \(P_1\).

If \(P_1(a, b)\) does lie in that plane, the line in which the plane intersects the \((x, y)\)-plane has the equation \(x = a\) (that’s how coordinates work in the plane). Looking at things from the point of view of \(P_2\), we get the equation \(x = c\) for the corresponding line. Since the two lines do intersect on the \(x\)-axis, we conclude that \(a = c\).

Thus, it remains to see that \(P_1\) lies in that plane. Indeed, the whole line between \(P\) and \(P_1\) lies in that plane. This is because that plane is perpendicular to the \((x, y)\)-plane and contains \(P\), so it contains the projecting segment.