Six-dimensional?

The following came up in the 12:00 lecture on Wednesday Sep 11.

I had mentioned that dot-product makes sense in any number of variables (whereas the cross-product was only for three). I wanted to give a “model” for a space of dimension greater than 3. So I mentioned the idea of talking about the location of two independently moving particles. The position of each has three variables, I said, and thus the two together need 6.

Let’s call the particles $A$ and $B$. To be suggestive, let $(x_A, y_A, z_A)$ be the coordinates of $A$, and $(x_B, y_B, z_B)$ be the coordinates of $B$. The combined list,

$$(x_A, y_A, z_A, x_B, y_B, z_B)$$

gives a point in a 6-dimensional space. The fact that the particles are both inside the same 3-dimensional space is recognized by taking the two projections of six variables onto three, viz., to $(x_A, y_A, z_A)$ and $(x_B, y_B, z_B)$.

If the two are moving, according to parametric equations:

$$(*) \quad x_A = x_A(t), \ y_A = y_A(t), \ z_A = z_A(t) \quad x_B = x_B(t), \ y_B = y_B(t), \ z_B = z_B(t).$$

One could store all of this as six equations for the two particles combined, or consider the two projections that separate the two particles’ data, as indicated in $(*)$.

If the above seems too mysterious, suppose we were looking at two particles moving on a line. Then one would track both of them in a plane, and you can see the two projections onto the line very clearly: Project onto one axis, project onto the other axis, and then think of the two axes as being the same (e.g., rotate one onto the other).