

### Assignment 3: due Thursday February 26

1. The proof of 1.19 on page 65 talks about parametrizing the two curves. On the other hand, the value of an integral does not depend on the parametrization. Is it possible to argue without parametrizations?

2. Let  $S(G)$  be the set of all simple closed rectifiable curves  $\gamma$  such that the closure of the region inside  $\gamma$  is contained in the region  $G$ . Can one obtain that

$$\int_{\gamma} f(z) dz = 0$$

for all  $\gamma \in S(G)$  from its truth for all triangles in  $S(G)$  by application of the approximation lemma?

3. We use the standard notation:  $z = x + iy$ ,  $f = u + iv$ .

a) Let  $G$  be an open set in the plane. Show that if  $f$  is complex analytic and real-valued,  $f$  is constant.

b) Give an example of a real-valued  $C^{\infty}$  function defined on the entire plane for which  $|f(z)|/|z^d| \rightarrow 1$  as  $|z| \rightarrow \infty$ .

4. Let  $\gamma_0$  and  $\gamma_1$  be simple closed curves in the plane, with  $\gamma_1$  inside  $\gamma_0$ .

a) Express that  $\gamma_1$  be inside  $\gamma_0$  analytically.

b) Describe *without referring to a picture* the breaking up of the region between the two curves into two Jordan regions.

5. From the textbook: page 87 – #6, 8, 9, 10.