

Nowadays, there is dissatisfaction everywhere in the nation over the inadequate learning of calculus by undergraduates. In my earlier positions as faculty at state universities, it was frequent lunchtime conversation to complain about one's calculus students. I'm not sure how my colleagues acted upon that griping, but most likely they were, like me, engaged in the activity of self-support that I now call *bitch-and-run*. It is better to follow through: is there anything one can *do* to raise the level of performance of the students? The ideal answer would be: change the attitude toward their education that too many students bring to college. However, it would take a miracle to get those students to discard immediately the view that their education is imposed from the outside, something to score "points" toward, to maneuver around, or to skim on.

Something we have experience with is that, when aspirations are fixed, notably when courses involve several instructors and common exams, there is little difference among the mean achievement levels of the students of each instructor. The students may say that one instructor explains things better than another, but that does not show in performance on the exams. One of my junior colleagues, a very clear lecturer, came to this realization independently; what, then, *is* the role of the instructor? she quite sensibly asked. (A cynical though plausible answer from a student viewpoint is: to minimize the time it takes to achieve that mediocre level of comprehension.)

Since giving better explanations does not improve learning of itself, we should be looking beyond that. I would not want science and engineering students at Johns Hopkins to take calculus through group learning in the classroom, for instance, but at least the calculus reformers are trying to address the issue. I maintain that through more or less traditional instruction of calculus, it is possible to substantially increase the students' command of the subject, without making unreasonable demands on them. Indeed, I believe I am doing that in Calculus for Engineering and the Physical Sciences. There are possibly other ways to attain such results, but it is not for me to guess what they might be. I stress that I would not be doing *exactly* what I do at Hopkins with the students at my previous universities, but the same general principles apply.

Most of us have been following fixed ideas about what we can and cannot do in a math "service" course, and much of that involves tolerating student reluctance to overhaul high school notions of learning mathematics. For instance, if you don't insist that freshmen read the textbook, most of them will be quite happy; if you force them to do that to the extent that they are capable of—most of them don't know what that is at the outset—many students will be unhappy. It is, after all, human nature to resist new exertion. *Yet by taking away the productive use of the textbook, one confines the students, depriving them of the principal means of developing skills for grasping the subject.*<sup>1</sup> A possible answer to my colleague's question is: the role of the instructor is to guide the student in reading the textbook, and to communicate a point of view on the material.

Many of us already do demand that the student read the book, but that is only the beginning. Here are the main points.

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<sup>1</sup>I once heard someone react that most of the textbooks are bad; therefore, I recommend using one of the good ones.

0. There must be a substantive presentation on mathematics and the sciences (combined) during Freshman Orientation. It must address the large change in aspirations that typically occurs in these disciplines when the student comes to college.

It is not at all obvious what to tell one's students that might help in bringing them up to college-level aspirations. The burden of guessing should be removed from the shoulders of the individual instructor. A presentation that seems to work is described in my article, "Telling the Truth." The instructor can repeat or modify that as he or she sees fit, but there will be a solid foundation.

1. The course is to be run at a level where the vast majority of the students are capable of learning the material reasonably well through reasonable exertion. (What students do with their time, though, is beyond our control.) Two hours outside of class per hour of class is not unreasonable. I'm aware that half of that is often regarded by students as a heavy workload, but if the students are to learn better, they will have to put in sufficient time. The instructor should be supportive but firm.

2. The professors of science and engineering want their students to be capable of using the calculus in their courses. We should uphold their wishes for flexible learning, and tell the students we are doing so. Do not accept as understandable the assertion by students that they don't want to learn the material, that they don't need it. I recommend, with a straight face, to such students that they drop the course, knowing fully well that the course is required for their intended major. *Why* is it required? I ask them. (Do they really want to declare themselves to be untrainable?)

3. We want students to learn methods, not problem types. This again is a change from what most of the students are accustomed to from high school. Stress the importance of conceptual material, as well as the working out of a large range of problems.

4. A bottom-line point about education in college (as opposed to high school) is that *the student* is responsible for learning the material, and is expected to cooperate in the learning process. Avoid retreating too much from this in your college course.

5. The students are capable of picking up the easier material of the course largely on their own. *This is of fundamental importance.* In particular, I'm saying that the students can learn some things from reading the textbook, even if they never had to do that in high school. The main detail is to determine how much is reasonable for students at *your* college. Don't just ask the students!

Refuse to waste a lot of time in class over the easier material, for that is not where they need our help. If a student is unwilling to cooperate and he falters, it is not our responsibility. For instance, here is as easy a topic as I can think of from Calculus II: integration by parts. How many examples of this technique would you as instructor show the class? If there were seven examples of integration by parts worked out in the textbook, as there are in ours, I am saying that it should be an important consideration in your answer to that question.

6. Allow the textbook to enhance what is done in class. *It is not necessary to wait to see the students "get" the material in the classroom.* They have until the homework is due for that! Don't be afraid to remind them of this. The students must be pushed to figure some things out for themselves, to become less dependent

on the instructor.<sup>2</sup> There is much more to a college course than a teacher feeding the students in class. Remind the students that learning is not a one-step process.

7. Many good students recommend looking a little at the material in the book before the lecture, so as not to go to class cold. I recommend that too. If they don't want to cooperate with this, they can sort things out later. It is a mistake to let students convince you that because they refuse to cooperate, you must go slower and thereby cover less.

8. Make sure that the exams test the students thoroughly. At Hopkins, I feel there is time for only two midterms and the Final. With class periods of only 50 minutes, it is hard to test them adequately at the regular class time.<sup>3</sup> That is one of three good reasons to give midterms in the evening. One can make the exams a little more thorough, and at the same time remove much of the time crunch for the students. True, it does introduce a few legitimate conflicts with the exam time, but most of them are easy enough to accommodate. I have settled on a 85-minute duration for the midterms.<sup>4</sup> Announce as excluded only topics that would not be on any reasonable exam you might give; ideally, there should not be too many of these.

Unfortunately, when you push students and cover the material in greater depth (even within the bounds of reasonable expectations) your ratings are very likely to go down.<sup>5</sup> In a situation where learning is the main issue, it is disappointing that so-called course evaluations barely pay lip-service to learning. An excuse for this is that it is more difficult to measure learning; it cannot be accomplished by polling students, for instance. So we ask for something simpler to determine, for something incomplete and less important, for just some useful feedback. Then we fail to remember, to a large extent, that is all we did! Since nothing else has been put forth, it can become by default that the way our educational function is assessed is by student reponse in the evaluations. It would make a lot of sense to see how students felt a year or two later, when they had some perspective, but that would be too inconvenient.

As always, such a set-up encourages the behavior that it is seen to reward. An easy way to elevate one's ratings is to cut material (that includes cutting depth) and explain more slowly that which remains—or does one trust the students to learn important harder topics on their own? I'm saying, in effect, that we must not allow this swindle to remain rewarded. Make sure that education, and not being the servant of student wishes, is what is being valued.

This is a large and annoying political problem, but it is not an issue at Hopkins, I'm happy to report. Not yet, anyway. But I have heard reports that in some universities, one's numerical scores on course surveys are put in as the teaching component of the formula for determining salary increments. Now, what does that encourage? And that instructors have been intimidated out of making even modest demands of their students for fear of bad evaluations. Some people even fear that giving bad grades will bring on poor evaluations. However, I don't believe that the latter is a serious issue at Hopkins. Not yet, anyway. But junior faculty do worry

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<sup>2</sup>A metaphor: the child must be weaned off the bottle.

<sup>3</sup>Sometimes there are two lectures in adjacent time slots. It is also difficult to get adequate space for an exam during regular hours.

<sup>4</sup>Then conflicts of 45 minutes or less pose no problem other than extended proctoring.

<sup>5</sup>My own ratings are good enough.

understandably that their careers would be jeopardized by bad evaluations. Is this our model of education?

One notion, which I find a bit misguided, is viewing our students as “customers”. If they *are* our customers, then what is it that they are buying (with their high tuition payments)? I can think of only two expected answers: an education, or a degree.<sup>6</sup> Note that in principle, either one is possible without the other. A modest proposal, then: Why obstruct the degree with courses, when we might simply sell the degree outright?... If the students are viewed, instead, as applying for our franchise, the nice analogy that H. Levine made in 1996, we have good reason to control the quality of our franchisees. Note, though, that in a highly commercialized society such as ours, these models will have the universities seeking to underbid each other, with nasty consequences in the long run. Is this our model of education?

Since I have advocated the determination of learning, I wish to relate a story pertinent to that. A past department chair, who felt I was overly concerned with working on student attitudes, pointed out that most people don’t do that.<sup>7</sup> He asked me to consider doing like a popular instructor, who gives “brilliant explanations.” I came back with “Does he get results?” That took my chair by surprise. “What do you mean?” he hastily asked. “Are his students *learning* better?” I continued. “I don’t know; how would one measure that?” I thought a little, and then said something like, “Well, it’s a bit difficult, but you might look at the exams to assess their difficulty, determine what the students knew in advance about the exams, and then look at the scores.” I am convinced that carefully effected aspirations do much more to improve learning than giving lectures that please most of the students.

I want to present an example of something I do, which some find exotic—most instructors don’t do it—to help students learn material on infinite series (the Engineering faculty here views all material on infinite series and Taylor series as extremely important). Even the suspicion that the assertion, “*The terms of the series go to zero, therefore the series converges,*” might be true will sabotage the student’s learning of convergence testing. Though the students will have seen counterexamples, a distressingly large portion of the class will make that statement on exams, even if you try to nail the point home in class. I therefore decided to impose a negative score penalty for making that mistake, and only for that mistake<sup>8</sup>, which I call *the ultimate sin*. I even tell the students the reason for the penalty. Still, it is passed from class to class that I am a mean instructor who does that. But in actuality, the penalty is a marvelous deterrent, more so for students who see their education in terms of external rewards. The mistake is almost completely eradicated, and the students know they have to do something else. Now that’s progress! The few students who do commit the error are out of touch and in trouble with the course anyway. I have recommended in writing imposing the ultimate sin penalty, and I suggest it orally to my colleagues. As far as I know, none of the latter seem willing to try it. Junior colleagues are convinced they can get the point across by good classroom presentation. See what I mean by fixed ideas?

In sum, you don’t get something for nothing. If you want your students to do better, then you have to push them to achieve. Be supportive but firm, and don’t

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<sup>6</sup>A third and better answer is: their classmates.

<sup>7</sup>I knew that they don’t. Still, I think it is the most important issue in undergraduate education, and that’s why I do.

<sup>8</sup>though I usually do the same for improper integrals

aim too low. It's routine to do that in athletics or musical performance. Most of us know already that we must persuade our students not to be mathematical "voyeurs", that you cannot improve your tennis game much by simply watching matches between professionals. Some students will be amazed how much they have learned; some students will eventually get reminded of that by other students, having grumbled about the instructor; some will act as though they had been poisoned.

If you prefer not to push your students, just kindly refrain from bitching.