For Problem Set #4

Prove or give a counterexample:

Let $f : \mathbb{R}^n \to \mathbb{R}$ be continuous at the point whose position vector is $\vec{a}$. Suppose that $D_{\vec{v}}(\vec{a})$ exists for all unit vectors $\vec{v}$, and satisfies

$$D_{\vec{v}}(\vec{a}) = \nabla f(\vec{a}) \cdot \vec{v}. \quad (1)$$

Then $f$ is differentiable at $\vec{a}$.

The above assertion is the converse of Theorem 6.2 on page 155.