Life in the electromagnetic lane

The laws of E & M are expressed in terms of the fundamental spacial first-order vector operators, namely divergence and curl, as well as derivative with respect to time. These are the equations of Maxwell, which contain some previously released material by other physicists. If you're wondering why we focus so much on *div* and *curl*, and not other operators, those (together with *grad*) are the ones that arise in physics, e.g. here. Surely we want to restrict ourselves to operators that are invariant (unchanged) when we compose with the rigid motions of $\mathbb{R}^3$.

The basic physical notions in Maxwell's equations are the *charge density* ρ (a scalar), and the vector quantities: *current density* $\mathbf{J}$, *electric field* $\mathbf{E}$, and *magnetic field* $\mathbf{B}$. The main point is that $\mathbf{E}$ and $\mathbf{B}$ are not independent; they are related by the third and fourth equations below (where I've made the normalization that the universal constants ϵ_0 and μ_0 equal 1):

$$\nabla \bullet \mathbf{E} = \rho, \qquad \nabla \bullet \mathbf{B} = 0, \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}.$$

These have practical consequences. For instance, the third equation implies that a changing magnetic field must be accompanied by a non-zero electric field. I seem to recall that the principle behind an electric motor lies inside these equations. Try to find it (anywhere)!