Homework 4

Sect 2.1 6, 8, 9, 16, 32, 34, 46

Sect 2.2 3 (but change the inequality to $0 \leq x^2 + y^2 < 4$),
6, 8, 14, 22, 30, 33, 36, 38, 42

and the following problem:

Prove the following “polar coordinate” criterion for limits—use $(\epsilon, \delta)$. Apply it to
\[
\frac{2x^2 + y^2}{x^2 + y^2} \quad \text{and} \quad \frac{2x^3 + y^3}{x^2 + y^2}.
\]

Proposition. Let $f$ be a function defined in a deleted neighborhood of the origin, and write it in polar coordinates $(r, \theta)$. Suppose that there is a function $g(r)$ such that $\lim_{r \to 0} g(r) = 0$ and
\[
|f(r, \theta)| \leq g(r).
\]

Then $\lim_{(x, y) \to (0, 0)} f(x, y) = 0$. 