## Mixed partials and differentiability

Concerning differentiability, I'm leaving it to you to appreciate the small notational changes that accompany dealing functions of the form $f : U \to \mathbb{R}$, where $U$ is an open set in $\mathbb{R}^n$ $(n > 2)$. If you wonder why the condition "open" is being imposed, you should recognize that it says that none of the points of $\mathbb{R}^n$ that lie on the boundary of $U$ are actually in $U$; that for each point of $U$, approaching it from all directions can be done entirely within $U$. The latter is important when we are taking limits, and we've been doing plenty of that!

We have seen why having continuous first order partials implies differentiability. Well, the proof that the continuity of the second order partials implies that the mixed partials are equal, is remarkably similar. It uses the Mean Value Theorem to bring in derivatives, and continuity to yield the equality. One makes clever use of the following obvious assertion about the so-called *double-difference*: *Let* $P_0$, $P_1$, $P_2$, *and* $P_3$ *be points in the domain of a function* $f$. *Then* $f(P_3) - f(P_2) - f(P_1) + f(P_0)$ *equals both* $[f(P_3) - f(P_2)] - [f(P_1) - f(P_0)]$ *and* $[f(P_3) - f(P_1)] - [f(P_2) - f(P_0)]$. See the textbook for the complete proof, where one expression leads to $f_{xy}(a, b)$ and the other—by interchanging the roles of the variables—produces $f_{yx}(a, b)$.