Inquiry 7: The Mean Value Theorem(s)

“The difference between science and the arts is not that they are
different sides of the same coin, or even different parts of the
same continuum, but rather they’re manifestations of the same
thing....They spring from the same source. The arts and sciences
are avatars of human creativity.”

— Mae Jemison, 1956–

You will discuss the following Inquiries with your group on Wednesday, 10/19. Please choose one person from each group to be prepared to present on these on Thursday, 10/20 (if you do not finish preparing in class, you may have to meet with your group outside of class). Finally, everyone in the class will be responsible for (individually) writing up their solutions to the problems below and submitting them on Monday, 10/24.

1. A function $f$ is said to be one-to-one (or injective) on an interval when for any two points $a$ and $b$ in the interval, $f(a) = f(b)$ implies $a = b$. (Said another way, $f$ cannot send two different points to the same place. In other words, two different input cannot correspond to the same output. Each output has a unique input.) Prove that if $f'(x) > 0$ for all $x$ in an (open) interval, then $f$ is one-to-one on that interval.

2. The Cauchy Mean Value Theorem is a stronger version of the Mean Value Theorem. It says that for two functions $f$ and $g$ which are continuous on $[a, b]$ and differentiable on $(a, b)$, there exists $x \in (a, b)$ such that

$$\frac{f'(x)}{g'(x)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

Prove it! (Hint: take a look at the proof of the MVT. Is there a clever choice of function to which you can apply Rolle’s Theorem that yields the CMVT? Start by moving $g'(x)$ over to the other side of the equation above. Then try to ‘undifferentiate’ it.)