Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ___________________________ Date: __________________

- This is a 50 minute closed book exam. No notes, books, or calculators are allowed.

- Present your solution to each problem in a clear and orderly fashion. Show all your work. An answer without justification will not receive full credit.

- Do not use any techniques we have not covered in class yet.

- This exam contains 6 pages (including this cover page) and 5 questions. The last page is intended for use as scrap paper.

The table on the right is for grading purposes. Please do not write in it.

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1. Determine whether each one of the following is TRUE or FALSE. If the statement is false, explain why or give a counterexample.

(a) (5 points) The function $f(x) = \sin(x)$ is one-to-one.

(b) (5 points) If a function $f(x)$ is continuous at $a$, then it is differentiable at $a$.

(c) (5 points) If $f$ and $g$ are differentiable functions, then $(fg)' = f'g'$.

(d) (5 points) If $f$ is a continuous function and $\lim_{{x \to 4}} f(x) = 5$, then $f(4) = 5$.

(e) (5 points) If $f$ and $g$ are continuous functions, then $f + g$ is a continuous function.
2. Evaluate the following limits justifying each step.

   (a) (15 points) \[ \lim_{x \to 2} \frac{x^2 - 4}{x^2 - 6x + 8} \]

   (b) (15 points) \[ \lim_{x \to \infty} \sqrt{x^2 + 3x + 1} - x \]
3. (a) (15 points) Let \( f, g \) be continuous functions defined on \([1, 3]\) such that \( f(1) < g(1) \) and \( f(3) > g(3) \). Show that there exists a number \( c \) in the interval \((0, 3)\) such that \( f(c) = g(c) \).

(b) (15 points) Find all asymptotes (horizontal and vertical) of \( \frac{\sqrt{2x^6 + x^4 + 3}}{x^3 + x^2 - 2x} \).
4. (a) (15 points) Find the derivative of \( f(x) = \sqrt{x^2 + 3} \) using the definition of derivative.

(b) (10 points) Find the equation of the tangent line to the graph of \( f \) at the point \((1,2)\).
5. (a) (15 points) Find the derivative of \( f(x) = \frac{x^2 + 2x + 1}{x + 2} \)

(b) (15 points) Find the values of \( A \) and \( B \) such that the function

\[
\begin{cases} 
  x^2 + 1 & x < 0 \\
  A \sin x + B \cos x & x \geq 0
\end{cases}
\]

is differentiable.
This page is intended for use as scrap paper.