Math 108  Midterm 2 Practice

Print Name: ________________________________  Section: ________________

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ________________________________  Date: ________________

- This is a 50 minute closed book exam. No notes, books, or calculators are allowed.

- Present your solution to each problem in a clear and orderly fashion. Show all your work. An answer without justification will not receive full credit.

- Do not use any techniques we have not covered in class yet.

- This exam contains 6 pages (including this cover page) and 5 questions. The last page is intended for use as scrap paper.

The table on the right is for grading purposes. Please do not write in it.

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1. Determine whether each one of the following is TRUE or FALSE. If the statement is false, explain why or give a counterexample.

(a) (5 points) If \( f(x) \) and \( g(x) \) are two differentiable functions such that \( \lim_{x \to 0} \frac{f'(x)}{g'(x)} \) exists, then the L’Hospital’s rule applies and \( \lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)} \).

(a) False

(b) (5 points) If \( g(x) \) and \( h(x) \) are two functions defined on an interval \((a, b)\) such that \( g'(x) = h'(x) \), then \( g(x) - h(x) \) is constant.

(b) True

(c) (5 points) If \( f \) is a continuous function on a closed interval \([a, b]\), then \( f \) attains an absolute maximum.

(c) True

(d) (5 points) If \( f(x) \geq 5 \) for \( x \) in \([1, 3]\), then \( \int_{1}^{3} f(x) \, dx \geq 10 \)

(d) True
2. (30 points) A poster is to have area 54 in$^2$ with 1-inch margin at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?

Answer

$6 \times 9$
3. Let \( f(x) = 2x^3 - 9x^2 + 12x - 1 \).
   
   (a) (10 points) Find the intervals on which \( f \) is increasing and the intervals on which \( f \) is decreasing.

   **Answer**
   Increasing on the intervals \((-\infty, 1), (2, \infty)\).
   Decreasing on the interval \((1, 2)\).

   (b) (10 points) Find the intervals on which \( f \) is concave up and the intervals on which \( f \) is concave down.

   **Answer**
   Concave down on the interval \((-\infty, \frac{3}{2})\).
   Concave up on the interval \((\frac{3}{2}, \infty)\).

   (c) (10 points) Find all critical points of \( f \) and indicate if they are local maxima, local minima, or neither.

   **Answer**
   Critical numbers are \( x = 1, 2 \). The critical number \( x = 1 \) is a local maximum, the critical number \( x = 2 \) is a local minimum.
4. (a) (15 points) Find the left Riemann sum approximating \( \int_{1}^{2} x^2 \, dx \) with 4 terms. You do not need to simplify.

\[
\frac{1}{4} \left( 1^2 + \left( \frac{5}{4} \right)^2 + \left( \frac{6}{4} \right)^2 + \left( \frac{7}{4} \right)^2 \right)
\]

(b) (15 points) Find the area under the graph of \( f(x) = x^2 + x^3 \) between \( x = 1 \) and \( x = 2 \).

\[
73 \quad \frac{12}{12}
\]
5. Evaluate the following limits

(a) (10 points) \( \lim_{x \to 0} \frac{\cos\left(\frac{\pi}{2} - x^2\right) \ln(1 + x) - x}{x} \)

**Answer**

\(-2\)

(Hint: Apply L’Hospital’s rule twice.)

(b) (10 points) \( \lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{1}{e^x - 1} \right) \)

**Answer**

\(\frac{1}{2}\)

(Hint: Rewrite as \(\frac{e^x - 1 - \sin x}{(e^x - 1)\sin x}\) and apply L’Hospital’s rule.)

(c) (10 points) \( \lim_{x \to \infty} e^x \sin \left(\frac{1}{x}\right) \)

**Answer**

\(\infty\)

(Hint: Rewrite as \(\frac{\sin(\frac{1}{x})}{e^{-x}}\) and apply L’Hospital’s rule.)
This page is intended for use as scrap paper.