Math 109    Midterm 1 Practice

Print Name: _______________________________  Section: ________________

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: _______________________________  Date: ________________

• This is a 50 minute closed book exam. No notes, books, or calculators are allowed.

• Present your solution to each problem in a clear and orderly fashion. Show all your work. An answer without justification will not receive full credit.

• Do not use any techniques we have not covered in class yet.

• This exam contains 7 pages (including this cover page) and 6 questions. The last page is intended for use as scrap paper.

The table on the right is for grading purposes. Please do not write in it.

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1. (20 points) Compute the following integral

\[ \int_{1}^{3} x^2 \ln(2x) \, dx \]
2. (20 points) Compute the following integral

\[ \int \sec^4 x \tan^2 x \, dx \]
3. (20 points) Compute the following integral

\[ \int \frac{x^2}{\sqrt{4 - x^2}} \, dx \]
4. (20 points) Compute the following integral

\[ \int \frac{3x^2 + 4x + 2}{x^3 + 2x^2 + 2x} \, dx \]
5. (10 points) Does the following improper integral converge? If so, find its value.

\[ \int_{1}^{\infty} \frac{\ln x}{x^2} \, dx \]
6. (10 points) Suppose a function $f(x) \neq 0$ satisfies the differential equation $y' = (x^2 - 4) \cdot y$. Find the interval on which the function $g(x) = \ln |f(x)|$ is decreasing. (You do not need to solve the differential equation to answer this question!)
### Trigonometric Identities

**Pythagorean Identities**

\[
\sin^2 x + \cos^2 x = 1
\]
\[
\tan^2 x + 1 = \sec^2 x
\]
\[
\cot^2 x + 1 = \csc^2 x
\]

**Sum and Difference Formulas**

\[
\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A
\]
\[
\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B
\]
\[
\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}
\]

**Double Angle Formulas**

\[
\sin(2A) = 2 \sin A \cos A
\]
\[
\cos(2A) = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A
\]
\[
\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}
\]

**Half Angle Formulas**

\[
\sin^2 x = \frac{1 - \cos(2x)}{2}
\]
\[
\cos^2 x = \frac{1 + \cos(2x)}{2}
\]
\[
\tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}
\]

**Product Formulas**

\[
\sin A \sin B = \frac{1}{2} (\cos(A - B) - \cos(A + B))
\]
\[
\cos A \cos B = \frac{1}{2} (\cos(A - B) + \cos(A + B))
\]
\[
\sin A \cos B = \frac{1}{2} (\sin(A + B) + \sin(A - B))
\]

### Differentiation Formulas

\[
\frac{d}{dx} (\ln |x|) = \frac{1}{x}
\]
\[
\frac{d}{dx} (a^x) = a^x \ln a
\]
\[
\frac{d}{dx} (\sin x) = \cos x
\]
\[
\frac{d}{dx} (\cos x) = -\sin x
\]
\[
\frac{d}{dx} (\tan x) = \sec^2 x
\]
\[
\frac{d}{dx} (\cot x) = -\csc^2 x
\]
\[
\frac{d}{dx} (\sec x) = \sec x \tan x
\]
\[
\frac{d}{dx} (\csc x) = -\csc x \cot x
\]
\[
\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}
\]
\[
\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}
\]
\[
\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1 + x^2}
\]
\[
\frac{d}{dx} (\cot^{-1} x) = -\frac{1}{x^2 + 1}
\]
\[
\frac{d}{dx} (\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}
\]
\[
\frac{d}{dx} (\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2 - 1}}
\]
Integration Formulas

\[
\int \frac{1}{x} \, dx = \ln |x| + C \\
\int a^x \, dx = \frac{1}{\ln a} a^x + C \\
\int \ln x \, dx = x \ln x - x + C \\
\int \sin x \, dx = -\cos x + C \\
\int \cos x \, dx = \sin x + C \\
\int \tan x \, dx = \ln |\sec x| + C \\
\int \cot x \, dx = -\ln |\csc x| + C \\
\int \sec x \, dx = \ln |\sec x + \tan x| + C \\
\int \csc x \, dx = -\ln |\csc x + \cot x| + C
\]

\[
\int \sec^2 x \, dx = \tan x + C \\
\int \csc^2 x \, dx = -\cot x + C \\
\int \sec x \tan x \, dx = \sec x + C \\
\int \csc x \cot x \, dx = -\csc x + C \\
\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C \\
\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C \\
\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1} \left|\frac{x}{a}\right| + C
\]

Trigonometric Substitution

\[
\sqrt{a^2 - x^2} \implies x = a \sin \theta \\
\sqrt{a^2 + x^2} \implies x = a \tan \theta \\
\sqrt{x^2 - a^2} \implies x = a \sec \theta
\]