SOLUTIONS TO PRACTICE PROBLEMS:

1.1 #24) Since denominator is \((x + 2)(x + 1)\), domain is all reals except \(x = -1\) and \(x = -2\).

1.1 #60) \(f(-x) = -x| -x| = -x|x| = -f(x)\). So \(f\) is odd.

1.4 #12) \(\lim_{x \to -4} x^2 + 5x + 4 = \lim_{x \to -4} \frac{(x + 4)(x + 1)}{(x + 4)(x - 1)}\)
\[= \lim_{x \to -4} \frac{x + 1}{x - 1}\]
\[= \frac{-4 + 1}{-4 - 1}\]
\[= \frac{-3}{-5}\]
\[= \frac{3}{5}\]

1.5 #14) Let us go through the three conditions for continuity:

a.) \(f(1) = 2\)

b.) \(\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{1}{x - 1}\) does not exist (goes to \(\pm \infty\)).

1.5 #28) Since \(f(x)\) is merely a trig function when \(x < \frac{\pi}{4}\) and \(x > \frac{\pi}{4}\), we know that \(f\) is continuous there. So we only need to check when \(x = \frac{\pi}{4}\). Let us go through the conditions:

a.) \(f\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\)

b.) \(\lim_{x \to \frac{\pi}{4}^+} f(x) = \lim_{x \to \frac{\pi}{4}^+} \cos x = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\)

\(\lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{x \to \frac{\pi}{4}^-} \sin x = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\).

So limit exists.

c.) \(f\left(\frac{\pi}{4}\right) = \lim_{x \to \frac{\pi}{4}} f(x)\).

Thus, \(f(x)\) is continuous at \(x = \frac{\pi}{4}\) and hence is continuous everywhere.

1.6 #20) The degree of the numerator is less than degree of denominator. So the limit is zero.

1.6 #28) \(\lim_{x \to \infty} \frac{x^3 - 2x + 3}{5 - 2x^2} = \lim_{x \to \infty} \frac{x^3(1 - \frac{2}{x^2} + \frac{3}{x^3})}{x^4\left(\frac{5}{x^2} - 2\right)}\)

1
\[
\lim_{x \to \infty} x\left(1 - \frac{2}{x^2} + \frac{3}{x^3}\right)
\]
\[
= \lim_{x \to \infty} x - \lim_{x \to \infty} \frac{2}{x^2} + \lim_{x \to \infty} \frac{3}{x^3}
\]
\[
= (\infty) - 0 + 0
\]
\[
= (\infty)(-\frac{1}{2})
\]
\[
= -\infty.
\]

2.1 #4) First, we find \( f'(c) \) (in our case, \( f'(-1) \)). So

\[
\begin{align*}
f'(x) &= 6x^2 - 5 \\
f'(-1) &= 6 - 5 = 1.
\end{align*}
\]

Now, we plug into the tangent line formula

\[
y - f(c) = f'(c)(x - c).
\]

Here, \( c = -1 \), \( f(c) = 3 \), and \( f'(c) = 1 \). So

\[
y - 3 = x + 1,
\]

or

\[
y = x + 4.
\]

2.2 #18) The formal definition of the derivative is

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h},
\]

where

\[
\begin{align*}
f(x) &= 1.5x^2 - x + 3.7, \\
f(x + h) &= 1.5(x + h)^2 - (x + h) + 3.7 \\
&= 1.5(x^2 + 2xh + h^2) - x - h + 3.7 \\
&= 1.5x^2 + 3xh + 1.5h^2 - x - h + 3.7.
\end{align*}
\]

So

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{(1.5x^2 + 3xh + 1.5h^2 - x - h + 3.7) - (1.5x^2 - x + 3.7)}{h} \\
&= \lim_{h \to 0} \frac{3xh + 1.5h^2 - h}{h}
\end{align*}
\]
\[
\lim_{h \to 0} 3x + 1.5h - 1 = 3x - 1.
\]

2.5 #10) Here, we note that \(\sqrt[3]{1 + \tan t} = (1 + \tan t)^{\frac{1}{3}}\). So we can use chain rule to take derivative. Let

\[
g(t) = t^\frac{1}{3} \\
h(t) = 1 + \tan t.
\]

Then

\[
f'(t) = g'(h(t))h'(t).
\]

Now,

\[
g'(t) = \frac{1}{3}t^{-\frac{2}{3}}, \\
g'(h(t)) = \frac{1}{3}(1 + \tan t)^{-\frac{2}{3}}, \\
h'(t) = \sec^2 t.
\]

So

\[
f'(t) = (\frac{1}{3}(1 + \tan t)^{-\frac{2}{3}})(\sec^2 t).
\]

2.7 #8) In this case, our equation is \(y = \sqrt{1 + x^3}\). So, by chain rule

\[
dy = \frac{1}{2\sqrt{1 + x^3}}(3x^2)dx,
\]

or

\[
\frac{dy}{dt} = \frac{1}{2\sqrt{1 + x^3}}(3x^2)\frac{dx}{dt}.
\]

Right now, we have that \(x = 2\), \(y = 3\), and \(\frac{dy}{dt} = 4\). So we solve for \(\frac{dx}{dt}\):

\[
4 = \frac{1}{2\sqrt{1 + 2^3}}(3(2^2))\frac{dx}{dt} \\
4 = \frac{1}{2\sqrt{9}}(12)\frac{dx}{dt} \\
4 = 2\frac{dx}{dt} \\
2 = \frac{dx}{dt}.
\]

Thus, the \(x\)-coordinate is increasing at \(\frac{1}{2}\) cm/sec.
3.1 #38) We go through the four steps:

a.) First, we check where the derivative is zero. We have

\[ f'(x) = 3x^2 - 12x + 9. \]

So

\[
\begin{align*}
3x^2 - 12x + 9 &= 0 \\
x^2 - 4x + 3 &= 0 \\
(x - 3)(x - 1) &= 0 \\
x &= 1, 3.
\end{align*}
\]

We check the y-coordinates:

\[
\begin{align*}
f(1) &= 6, \\
f(3) &= 2.
\end{align*}
\]

b.) Next, we find where the derivative is undefined. This never happens.

c.) Now, we check the y-values for the endpoints:

\[
\begin{align*}
f(-1) &= -14, \\
f(4) &= 6.
\end{align*}
\]

d.) Finally, we compare the values. The absolute maxima occur when \( y = 6 \), which corresponds to \( x = 1 \) and \( x = 4 \). The absolute minimum occurs when \( y = -14 \), which is when \( x = -1 \).

3.2 #2) To show that the function satisfies Rolle’s theorem, we check the three conditions. First, it is clear that \( f \) is continuous and differentiable. So we check that \( f(a) = f(b) \):

\[
\begin{align*}
f(0) &= 5, \\
f(2) &= 5.
\end{align*}
\]

Now, we wish to find all values \( c \) for which \( f'(c) = 0 \). First,

\[ f'(x) = 3x^2 - 6x + 2. \]

To find when this is zero, we wish to solve

\[ 3c^2 - 6c + 2 = 0. \]
Quadratic formula tells us that this occurs when

\[ c = \frac{6 + \sqrt{36 - 24}}{6} = \frac{6 + \sqrt{12}}{6} = \frac{3 + \sqrt{3}}{3}. \]

5.4 #34) If \( y = (\sin x)^x \) then

\[ \ln y = \ln(\sin x)^x \]
\[ \ln y = x \ln(\sin x). \]

Taking the derivative of both sides (and using the chain rule on \( \ln \sin x \)):

\[ \frac{1}{y} \frac{dy}{dx} = x \frac{\ln \sin x}{\sin x} \cos x + \ln \sin x. \]

Dividing both sides by \( dx \) gives

\[ \frac{1}{y} \frac{dy}{dx} = x \left( \frac{1}{\sin x} \right) \cos x + \ln \sin x. \]

Solving for \( \frac{dy}{dx} \), we multiply both sides by \( y \) to get

\[ \frac{dy}{dx} = \left[ x \left( \frac{1}{\sin x} \right) \cos x + \ln \sin x \right] y. \]

Plugging \((\sin x)^x\) back in for \( y \), we have

\[ \frac{dy}{dx} = \left[ x \left( \frac{1}{\sin x} \right) \cos x + \ln \sin x \right] (\sin x)^x. \]