HOMEWORK 2

Chapter 2-1: #4, 5.
Chapter 2-2: #4, 6.
Chapter 2-4: #4, 5.

Also, do the following problems:

Problem 7: Let \(2 = p_1 < p_2 < p_3, \ldots\) be the sequence of primes in increasing order. Prove that 
\[ p_n \leq 2^{2^n-1}. \]
(Hint: Show that the method used to prove Euclid’s Theorem also proves that 
\( p_n \leq p_1p_2\cdots p_{n-1} + 1 \).)

Problem 8: Prove that there are arbitrarily long intervals of composite numbers. In other words, for any \(n \in \mathbb{N}\), prove that there exists an integer \(a\) such that 
\(a, a + 1, a + 2, \ldots, a + (n - 1)\) are all composite. (Hint: Consider the expression 
\((n + 1)! + k\). For which \(k\) must this be composite?)

Challenge Problem: Prove that if \(n \neq 2^k\) for any \(k \in \mathbb{Z}\) then \(p = 2^n + 1\) is not prime.