SHORT RESPONSE TO TUNIS'S LETTER TO THE EDITOR ON TECHNOLOGY IN COLLEGE

I have been invited to respond to the letter of Harry B. Tunis (*Educational Studies in Mathematics*, Vol. 59, 2004,) which requests advice about how to best insert technology into mathematics education at the college level. The questions he asks need a context and that gives me the welcome opportunity to talk about mathematics education in general and the articulation between K-12 and college in particular. I thank the editor for this opportunity.

Let me begin by describing my part in the grand scheme of mathematics education. Like almost all who do research in mathematics I pay my debt to society by teaching. However diverse our research interests in mathematics are, my colleagues from around the United States, and, indeed, from around the world, almost all teach college undergraduates their basic mathematics. There are, regrettably, very few mathematics majors. Almost all of our students are first and second year college students. They are not really "our" students though. They are majoring in other departments and those departments have required them to take specific mathematics courses. These departments make it quite clear to the mathematics departments what material they want covered. We, in mathematics departments, have very little say about the curriculum for the vast majority of our students. We run "service" departments.

I have been a bit rigid in my story for the sake of efficiency. Numerous qualifiers should be put in. Of course we talk and negotiate with the departments we service. Of course we have some flexibility in what we do, especially with terminal courses like pre-med calculus. Such qualifiers do not alter the main point: the mathematics curriculum is determined from the top, not the bottom. We, in college mathematics departments, like those in K-12, are not at the top.

Because we must teach specific material to other departments' students, we have to start with students who have adequate preparation to learn that material. This makes us "experts" on what students need to learn in K-12 since that is where our students have just come from. To determine students' level of preparation, most colleges give placement exams. Students who do not pass the placement test for a college level mathematics course rarely finish a year of college level mathematics. Consequently, if a student does not go to college prepared for college mathematics then all of the career choices that college mathematics offers are closed to the student.

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It is important to realize that mathematicians have little control over what they teach. That is not decided by us but by the departments that need the mathematics. Things are really no different in K-12 if the students are to be prepared for college. Now, back to the issue. The departments that determine what we teach are not requesting the insertion of technology into our courses. If it was important, they would let us know and we would do it. This gets back to the statement in Tunis's letter: "Students at selective colleges might even wonder why their college courses use less technology than their secondary school experiences." The student who gets the point will turn this statement around: "Students at selective colleges might well wonder why their secondary school experiences used more technology than their college courses." And they would be right to question this. The mathematics curriculum is determined top down, starting with the end user, and college mathematics teachers are caught somewhere in the middle of this pipeline. What we teach in college is not and cannot be determined by what is taught in K-12. If students are not taught the proper things in K-12, their mathematical career ends in K-12.

So, why is there so little technology in those first 2 years of college mathematics? Well, first, our clients are not asking for it, and, the students are not our clients, the other departments are our clients. Second, and perhaps more to the point, I have not yet encountered a mathematics concept that required technology to either teach it or assess it. The concepts and skills we teach are so basic and fundamental that technology is not needed to either elucidate or enhance them. There might be teachers who can figure out a way to enhance learning with the use of technology, but it is absolutely unnecessary. Consequently, all of Tunis's questions about how best to insert technology into these introductory courses in college are really a non-issue. The courses are fairly hard and move fast. Introducing extraneous and unnecessary technology would divert attention from the goals of developing a basic understanding and problem solving skills with the new material. Do not take a minute away from that to use technology. This is what we want students to learn and so we get back to Tunis's comment: "In short, I'm not sure we can give the same kinds of exams that we have traditionally given." Actually, the traditional exams are designed to test the students' understanding and problem solving ability with the new material. The true test of whether technology in the classroom has succeeded is whether it improves the students' performance on traditional exams. YOU CANNOT REDEFINE MATHEMATICS!

Having discussed the lot in life of a college mathematics professor and some of our thoughts on technology, let me move on to talk more about the articulation issue brought up by Tunis and what this implies for the use of technology in the K-12 classroom. Recall that since 1990, over 60%

of all high school graduates in the United States have gone on directly to college (e.g. http://nces.ed.gov/programs/digest/d02/tables/dtl84.asp). Around 75% go on within two years of graduating from high school. College preparation is thus the default choice for K-12 education. This 75% should be prepared to pass their college placement test and proceed with college mathematics. If they are not prepared, their career choices are dramatically limited. These career choices are limited for them by their K-12 experience if they do not have the opportunity to get prepared for college in K-12.

The good news is that there is very little mathematics that must be mastered in K-12 to be prepared for college. Students need to know basic arithmetic by which I mean the 4 operations with whole numbers, decimals and fractions. They need to know ratios, proportions and percent. Underlying this is the base 10 place value system. They need to know some measurement and basic geometry facts. That, and using those skills at problem solving, pretty much takes care of K-6 or 7. After that they need two years of a good algebra course and it is desirable that they have some real Euclidean geometry. I have skimped of course by not specifying what should be taught in algebra but this is not the place.

This is, however, an opportunity to tell K-6 teachers how very important what they teach is. These are the foundation years and, if students do not learn the foundations, they seldom recover. I can imagine that it might sometimes be difficult for K-6 teachers to see the connection between what they teach and what their students will need in college but the connection is there and very strong.

Now I get into touchy territory. I am well aware of how the so-called standard algorithms for arithmetic have fallen on hard times. Educators frequently dismiss them as obsolete and unnecessary. On the contrary, from the point of view of the teacher who gets your students next, I cannot emphasize enough how important these basic algorithms (or their equivalent) are. There are many reasons for this and properly dealing with them would require a paper in itself. Briefly,

1. The standard algorithms are the only collection of beautiful, serious, mathematical theorems you can teach to a child in K-6. They are amazingly powerful. They take the *ad hoc* out of arithmetic. They extend observed patterns and give the operations structure. These theorems solve the age-old problem of how to do basic computations without having to use different strategies for different numbers. The mystery is gone.
2. Students will be confronted with new algorithms constantly as they progress in their study of mathematics. Ignoring the most basic and most important of all algorithms is not good preparation.

3. In high school and college mathematics, these very same algorithms will be slightly modified and generalized and used in different settings. This happens many times over and a mastery of the original algorithms makes this process an incremental one rather than an overwhelming one.

Given the thrilling description of the standard algorithms I have just relayed, I found Tunis's final comment disheartening: "How can we give them the idea that mathematics is more than the rote learning of algorithms?" Again, I would turn this around and ask "How could anyone get the idea that mathematics is just the rote learning of algorithms?" If the standard algorithms are taught with understanding, enthusiasm and admiration then a student should find them exciting and appreciate the awesome power they give.

Where, if anywhere, do calculators come into this picture? At the K-6 level there is NO PLACE for calculators. I am not alone in this belief. There is a listserve for a few hundred research mathematicians who specialize in algebraic topology (my field of research). These colleagues are from all over the United States and the world. They teach at every kind of college from community colleges to the elite private universities. Some of them are now deans or in industry (but stay in touch through this listserve). It is a great cross section of mathematicians. These colleagues get K-12 students next. If you ask them what mathematics is, you will get a raging debate with little or no agreement. However, I asked them if they agree or disagree with:

> In order to succeed at freshmen mathematics at my college/university, it is important to have knowledge of and facility with basic arithmetic algorithms, e.g. multiplication, division, fractions, decimals, and algebra (without having to rely on a calculator).

I had 93 responses in support of this and no responses opposed. Even I was amazed as some of my colleagues usually disagree just to be contrary. This, however, is important to them all. Many of them made comments which are enlightening and worth reading (http://math.jhu.edu/~wsw/ED/list). Of the shorter ones, we have: "I am shocked that there is any issue here." "That it is even slightly in doubt is strong evidence of very distorted curriculum decisions." "I am sorry to see your talent wasted on what I recognize as a battle over mathematical competence." "It is sad that such things which ought to be completely obvious are controversial." From a Japanese colleague in Nagoya: "I thought it was a joke for you to have asked our opinion about such a self-evident truth, but I am afraid I was wrong. I am very sorry that you had to do this."

The shock that many displayed is an indication of how little involvement there is of research mathematicians in the development of the K-12 curriculum. I have been involved for only a few years. One area I have been quite involved with is the various states' written standards for K-12 mathematics education. These standards are what guide the teachers and tell them what is important to teach. Many states have so many topics each year that the all important core of arithmetic does not stand out. Many states never mention dividing fractions and long division. Few mention the standard algorithms. To the point of this note, many states insert technology starting in kindergarten or first grade. Many other states are (deliberately?) ambiguous about the use of technology. Such standards indicate a significant disconnect from the pipeline that leads to college mathematics and real career choices. There should be continuity. As explained above, college teachers cannot "adjust" college level mathematics courses to suit inappropriate preparation in K-12. K-12 is preparation for college. There seems to be a breakdown in communication about articulation between K-12 and college. Perhaps this is just an artifact of the time when the majority of students did not go to college and the articulation issue was not so important. However, the time has come for more college mathematics professors to be included in the development of the K-12 mathematics standards and curriculum. Without our input the pipeline may stay disconnected.

I have emphasized K-7 mathematics education here because I wanted to stress its foundational importance due to its continuity with college level mathematics; 8–12 is a whole other (time consuming) world which also depends entirely on good K-7 preparation.

There is, in some circles, tremendous resistance to the importance of pencil and paper calculation. But, the hands on, pencil and paper grappling with mathematics is essential for the mathematical maturation process that is necessary for the preparation for college level mathematics, in particular, for preparing the groundwork for real mathematical reasoning and problem solving. Perhaps it is not necessary for those few who do not go on to college, but for the majority who are college bound it is. Students must be given the opportunity to learn the foundations of mathematics so they can have the career choices they should when they go to college.

We, in college, frequently hear criticism such as Tunis's. Many K-12 educators tell us we are behind the times, that the standard algorithms are obsolete and calculators should be used for everything. We are told to stop living in the past and adapt to the new world. If and when we hear that from the departments we serve, we will adapt. As it is, we are pretty clear on what mathematics is and what mathematics a student must know coming out of K-12 in order to have a chance at succeeding in college mathematics courses. I believe that K-12 educators should be listening, or at least talking, to us more.

In summary, Tunis's questions indicate a lack of understanding of what mathematics is and what students need to learn about mathematics. To be sure, technology is important, but it is a separate subject; it is not mathematics. Success in college depends not on the insertion of technology but on the hard work of K-12 teachers, building a solid foundation with which we can work.

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