Washington State
High School Math Text Review

by

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A few basic goals of high school mathematics will be looked at closely in the top programs chosen for high school by the state of Washington. Our concern will be with the mathematical development and coherence of the programs and not with issues of pedagogy.

Algebra: linear functions, equations, and inequalities

We examine the algebraic concepts and skills associated with linear functions because they are a critical foundation for the further study of algebra. We focus our evaluation of the programs on the following Washington standard:

A1.4.B *Write and graph an equation for a line given the slope and the y-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.*

We also consider how well the programs meet the following important standard:

A1.1.B *Solve problems that can be represented by linear functions, equations, and inequalities.*

Linear functions, equations, and inequalities in *Holt*

We review Chapter 5 of *Holt Algebra 1* on linear functions.

The study of linear equations and their graphs in Chapter 5 begins with a flawed foundation. Because this is so common, it will not be emphasized, but teachers need to compensate for these problems.

Three foundational issues are not dealt with at all. First, it is not shown that the definition of slope works for a line in the plane. The definition, as given, produces a ratio for every pair of points on the line. It is true that for a line these are all the same ratios, but no attempt is made to show that. Second, no attempt is made to show that a line in the plane is the graph of a linear equation; it is just asserted. Third, it not shown that the graph of a linear equation is a line; again, it is just asserted.

The failure to address these fundamental issues leaves the study of linear functions and their graphs on shaky ground.

On page 335 is a single problem that starts with y-intercept 3 and slope 2 and rigorously arrives at the equation y=2x+3. This example is the only justification for the "m" in y=mx+b being the slope. Identifying "m" with the slope is a quick and simple algebraic manipulation, but it is not done here, or in any of the other programs. Although the y-intercept is taken care of nicely earlier when the standard form of a linear equation is studied, the failure to connect the definition of the slope to the "m" in the slope-intercept form of a linear equation creates another foundational issue.

We also have:

> Any linear equation can be written in slope-intercept form by solving for y and simplifying.

Although this is stated, the computation is not carried out for the general case.

Ignoring these foundational flaws, the chapter begins with the standard form of a linear equation, page 298. Since it is assumed we get a straight line, it is enough to plot some points and draw the line in order to graph the equation. It is easy to compute the intercepts from the standard form. That computation gives two points on the graph and is enough to draw the graph.

Slope is introduced and worked with, in the beginning mostly from tables and reading from graphs and then with more algebraic techniques. The slope intercept form, page 335, is introduced and applied. Page 342 brings the point-slope form, including something that resembles a proof, if you accept their slope.

Inequalities in one variable are covered in Chapter 3, but multiplication of inequalities by negative numbers is weak. The number line is drawn and it shows a couple of points being flipped when multiplied by -2. This treatment of inequalities is inadequate.

Summary: Mathematical underpinnings are missing and multiplication of inequalities by negative numbers is weak. That said, the material in general is developed meticulously. There are numerous exercises and word problems, including exercises that require students to translate between the forms of linear equations and quite long collections of related problems.

Linear functions, equations, and inequalities in *Discovering*

Linear equations are introduced in *Discovering Algebra*'s Chapter 3 by plotting points recursively on a graphing calculator. On page 166, after demonstrating an example, the concept of *linear relationship* is vaguely defined:

> The points you plotted in the example showed a **linear relationship** between floor numbers and their heights. In what other graphs have you seen linear relationships?

"Linear relationship" means something mathematically, and the definition provided is hopelessly inadequate. However, *linear relationship* is used freely in the text from here on.

On page 179 we see our first linear equation, and it is put in what is called the **intercept form**, y=a+bx. The y-intercept is identified as the letter "a". The "b" term is computed numerically in several examples in Lesson 3.5. This is done in terms of **rate of change**. The examples given are tables and intended for calculator use. There are some good word problems that require the construction of a linear function.

Techniques for solving linear equations are introduced in Lesson 3.6. Algebraic techniques are introduced and so are calculator techniques. There are a couple of problems with the mathematics. They start an example solution on page 198 with the statement: "Each of these methods will give the same answer." This claim is not true as becomes apparent on the next page.

> From Example B, you can see that each method has its advantages. The methods of balancing and undoing use the same process of working backward to get an exact solution. The two calculator methods are easy to use but usually give approximate solutions to the equation. You may prefer one method to others, depending on the equation you need to solve.

This is a text for students to learn the algebra in order to meet the Washington State mathematics standards. The exact solution obtained using the algebraic technique referred to as "balancing" meets these standards. "Undoing" is just the list of buttons to push to solve the equation on a calculator. Giving three calculator solutions equal status to algebraic techniques undermines the goal of teaching students algebra.

On page 218 we find a formula for the slope of a line. The discussion in the text up until this point loosely identified slope with the "b" of the intercept form of a linear equation (see the question at the bottom of page 219). The formula defines the slope of a line as the ratio of the change in y to the change in x for two points on the line. No attempt is made to show that the same ratio is obtained if two different points are used. This discussion of the slope of a line is inadequate.

The **slope-intercept form** of a linear equation is introduced on page 229 in Lesson 4.2 on Writing a Linear Equation to Fit Data. **Line of fit** for a scatter plot is not formally part of this review, but since this section introduces the important slope-intercept form, a comment will be made. The line of fit in this lesson is determined by what "looks best." Appearance is most definitively NOT mathematics and should not be invoked. There are mathematical reasons that allow one to determine a line that fits the data of a scatter plot, but these reasons are not available to students at this level.

In Lesson 4.3 starting on page 234, the point-slope form for linear equations is nicely developed.

In Lesson 4.4 very basic algebraic manipulations are discussed in order to show that some linear equations are equivalent. As this begins only on page 240 of an algebra book, it shows how little the structure of algebra is emphasized. The standard form shows up and algebraic manipulations allow one to go between the various forms introduced so far: standard, intercept, slope-intercept, and point-slope.

There are many good word problems throughout the study of linear equations.

What is missing, as in *Holt Algebra I*, is the mathematical foundation. There is no attempt to show that a line in the plane is the graph of a linear equation and that the graph of a linear equation is a line. Moreover, these unproven assertions are never even highlighted. They are just there.

Single variable inequalities are studied in Lesson 5.5. Inequalities respond differently from equations when it comes to multiplying and dividing by negative numbers. This is never explicitly stated in this section. It is left to the student to discover it in an investigation.

Summary: The foundational necessities of mathematics are missing from the graphing of linear functions. The material is developed, but the emphasis is not on the structure of algebra and the importance of symbolic manipulation is minimized.

Linear functions, equations, and inequalities in *Core-Plus Mathematics*

The place to look for these topics is in Unit 3 of *Course 1*, pages 150-237. It is worth going through the text page by page to see how the mathematics develops. We will keep in mind the two standards of interest: A1.4.B and A1.1.B.

Immediately, on page 150, we are given the definition of a **linear function**. It is a function with straight-line graphs. We are then introduced to a linear function, B=20+5n, and this is analyzed. A major theme of the book shows up here. The analysis is done by creating a table and graphing the points from the table. Then we are given tables and asked to find a linear function. At no point is there an attempt to show that the equation's graph really is a line. Likewise, there is never an attempt to show that a line graph (i.e. coming from a linear function) comes from the usual form of a linear equation. This is a significant flaw in the mathematical foundation provided in the text.

On page 155, *Course I* discusses the mathematics underlying the problems it solves. Because the text's focus is on the problems, it refers to the mathematics as "Linear Functions Without Contexts". Mathematics, itself, can be considered a context, especially in a mathematics course. Here the text defines the slope of a linear function, and its proof that it is well defined (though they don't state any such concern) is that "You've probably noticed by now that the rate of change of a linear function is constant." This is another significant flaw in the development of the mathematical foundations.

On page 156 the text summarizes the mathematics:

> Linear functions relating two variables x and y can be represented using tables, graphs, symbolic rules, or verbal descriptions.

Although this statement is true, the essence of algebra involves abstraction using symbols. This description of linear functions accurately reflects what the text emphasizes, and, in particular, how it downplays the importance of the symbolic approach. There are, for example, many good problems in the text, but this statement illustrates fairly well the distribution of time spent on each of the four representations.

On page 157 we get another insight into the view this program takes of these simple linear functions.

> Mathematicians typically write the rules for linear functions in the form y=mx+b. Statisticians prefer the general form y=a+bx.

The text continues by using the form preferred by statisticians, not the one used by mathematicians. By itself this particular notational choice is not important, but because mathematics is slighted in so many ways, an odd, subtle, anti-mathematics tone pervades the program.

On page 160, the **slope-intercept** form is given a name. Also on this page, we have the only place we could find where a linear function is computed from two points in the plane. This happens in problem 6 with four cases.

The next section is not about algebra or mathematics at all. It is about drawing lines that "fit" the data of a scatter plot. The lines are drawn without benefit of mathematics, guided by words and phrases like "fits the data closely", a "line that you believe is a good model", "you believe the graph closely matches", etc. There are mathematical ways to do this, but they are quite a bit more advanced than can be done here, so mathematics is ignored.

Inequalities show up in a problem on page 191. On this same page we learn:

> It is often possible to solve problems that involve linear equations without the use of tables, graphs, or computer algebra systems. Solving equations by symbolic reasoning is called solving *algebraically.*

The text then demonstrates two ways of solving $30x+12=45$ algebraically. The first way is a straightforward sequence of algebraic manipulations. The second way gives the buttons you'd push to "undo" the equation to solve it on a calculator. The fact that this is considered solving the equation "algebraically", combined with the de-emphasis on symbolic work, is more evidence of how algebra is downplayed in this program.

The serious study of inequalities begins on page 194. A problem is given to see what happens with three numerical inequalities when you multiply by minus one. Based on this you are to decide what happens in general. No serious attempt at explanation is offered.

Finally, in Lesson 3, Unit 3, of *Course 1,* 214 pages into high school mathematics, the program takes on the concept of equivalent expressions and goes over the elementary ways of computing with simple algebraic expressions, such as distributivity, commutativity, associativity, and the inverse properties of addition/subtraction and multiplication/division. For linear expressions in one variable there is a canonical simplification, so some of the problems are a bit strange like when they ask, on page 219, for "at least two different but equivalent expressions" for 8x-3x-2x-50. Here we find another example of devaluing algebra when, two pages after introducing basic algebraic manipulations, the program has moved to a calculator with a computer algebra system.

Summary: This program has a multitude of good problems, but never develops the core of the mathematics of linear functions. The problems are set in contexts and mathematics itself is rarely considered as a legitimate enterprise to investigate. Although there is some minimal required algebraic manipulation all along, there are only two pages devoted to it, pages 220-221. This lack of attention to algebraic manipulation means that the form Ax+By=C never shows up and neither does the translating back and forth between forms. It is okay to develop the mathematics through problems as this program does, but then it is essential to consolidate the actual mathematics. Symbolic algebra is minimized. As important as problem solving is, students must move forward with the mathematics as well.

Linear functions, equations, and inequalities in *Glencoe McGraw-Hill*

The study of linear equations, functions, and inequalities is covered in *Algebra 1*, Chapters 2-5, pages 73-330.

Chapter 2 begins with a few pages that explain how to translate word sentences in word problems into mathematical equations. The text begins with things that are familiar and easy such as "The length of each lap times the number of laps is the length of the race" and moves on to more complex sentences such as "Seven times a number squared is five times the difference of *k* and *m*." It is nicely done.

The bulk of the material from this point in the book is a careful instruction on how to solve more and more complex linear equations. Linear equations aren't that complex to begin with, but we know students struggle with them nonetheless. Time is spent on each progressively more difficult type of equation, starting with examples like c-22=54, 63+m=79, and then moving on to equations of the type (2/3)q=1/2. Finally, we get equations of the form 11x-4=29. There are many exercises and word problems for each level of problem, including word problems that require combining terms algebraically to put the equation into one of the above forms. The work continues with equations that have the unknown on both sides, such as 2+4k=3k-6. A nice mixture of problems surrounds the instruction. There are problems on number theory, geometry, money, etc.

Some word problems are great, for example, on page 133:

> "Mrs. Matthews has 16 cups of punch that is 3% pineapple juice. She also has a punch that is 33% pineapple juice. How many cups of the 33% punch will she need to add to the 3% punch to obtain a punch that is 20% pineapple juice?"

Although the focus of the text is on algebra skills, solving such problems requires significant understanding.

Chapter 3 moves from linear equations to linear functions. The chapter begins badly with the statement, page 153:

> A linear equation is an equation that forms a line when it is graphed. Linear equations are often written in the form Ax+By=C.

Never is it indicated that there are hidden assumptions here. As with all the other texts under review, the chapter does not show that this is the necessary form of an equation that gives a line, nor does it show that this equation gives a line. The foundation of the study of linear functions is flawed.

The program compounds its problems with the truly bizarre "key concept" on page 153:

> The **standard form** of a linear equation is Ax+By=C, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1.

Such a statement would rule out any equation that had Pi or the square root of 2 in it. This is a place where the book could have benefited greatly from a good proofreading by someone much more familiar with mathematics than the authors. This example is particularly egregious, but there are other missteps, frequently when the authors try, as above, to be precise, and fail.

Having ignored the fundamental issues, they proceed to explain the graphing of linear functions with the same meticulous detail they used to explain the solving of linear equations. On page 173, slope is defined, but the usual problem of not bothering to show that it is well defined shows up in this text as well. The best the authors can do is to state on page 172: "Because a linear function has a constant rate of change, any two points on a line can be used to determine its slope". The issue is that it hasn't been shown that "a linear function has a constant rate of change".

In Chapter 4 we are finally introduced to the **slope-intercept** form of a linear equation, y=mx+b. Despite it being an easy calculation, the authors do not show that "m" in the equation is the slope; they just plug in the slope for m when they want. This is another of the usual foundational flaws.

On pages 224 and 225 the authors show how to find the equation of a line when given the slope and a point on the line or when given two points on the line. Formulas are not derived, but example problems are worked. Later, page 231, the **point-slope** form is given as a formula without justification. Given the derivation of this formula for example cases, the lack of justification is unnecessary. Indeed, the next page tells you exactly how to do it, as well as how to find the equation when you are given 2 points. The general form is not done though.

Throughout there are examples of moving from one form to another, sometimes just to solve a problem, and other times explicitly for that goal, such as on page 233 to go from point-slope form to standard form.

Lesson 4-5 exhibits the usual non-mathematical "lines of fit", i.e. from page 246: "Draw a line that seems to pass close to most of the data points."

Chapter 5 is devoted to linear inequalities. The authors only state the reversal of inequalities that comes from multiplying by a negative number. No attempt is made to demonstrate this is true except for the simple example of multiplying 7<9 by -2 and observing the results. This treatment is quite inadequate.

Summary: With the exception of multiplying and dividing inequalities by negative numbers, the material of the standards is all developed extremely well in an orderly progression from the simplest of equations to the more complex issues with functions. The focus is on the mathematics and how it is used for problem solving. The crucial element here is that the mathematics is not a secondary item in this program. However, the text either ignores or is oblivious to all of the foundational issues associated with the graphing of linear functions. In addition, this program suffers from the mathematics not having been proofread carefully.

Summary of linear functions, equations, and inequalities

All four programs have ignored the basic foundational issues in the development of the graphing of linear functions, so comparing them on those issues is irrelevant. Supplementation by the teacher will be essential for understanding.

Both *Holt* and *Glencoe* develop the structure of the subject very well, but they do it in quite different ways. In each case the focus is on the math and its applications. *Glencoe* has a better set of problems, but abuses mathematical language and precision far too much. Consequently, how to rate *Glencoe* is a quandary. It is both good and bad, and so is an outlier in this review. However, to this reviewer, *Glencoe* still seems the best in this category.

Both *Discovering* and *Core-Plus* minimize the importance of the core of algebra in their texts. It is disconcerting to see pictures of graphing calculators on so many pages in *Discovering* and to have computer algebra systems offered up in *Core-Plus*. The focus of each of these texts is not on the development of the algebra for linear functions. That is secondary to their purposes at best and consequently obscures the underlying structure of the mathematics, more so in *Core-Plus* than in *Discovering*. Both have good collections of problems, though. *Discovering* is ranked above *Core-Plus* but both are unacceptable texts.

Algebra: max/min problems for quadratics

The Washington state standards for algebra focus on problem solving. The ability to put quadratic functions in vertex form allows students to use symmetry and to find the maximum or the minimum of the function. This opens up a new world of problems the student can solve, namely max/min problems. The approach to max/min problems will be analyzed for both the basic algebra and the conceptual development and to be sure that the connections among strands and topics are explicit and make good sense. The standards we use as our guideposts are:

A2.3.A *Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.*

A2.1.C *Solve problems that can be represented by quadratic functions, equations, and inequalities*.

Max/min problems for quadratics in *Holt*

Although we focus on max/min problems, and these are fairly advanced, it is worth taking a quick look back at Chapter 9 in *Algebra 1*, where quadratic functions and equations are introduced, and then move on to Chapter 5 of *Algebra 2*, where they are wrapped up.

Preceding Chapter 9, part of Chapter 7 develops the arithmetic of polynomials and Chapter 8 works on factoring. Having these skills covered before the study of quadratics makes it much easier to work with them.

Algebra 1 and *Algebra 2* define quadratic functions differently. On page 590 of *Algebra 1*, the definition of a quadratic function is one "that can be written in the standard form $ax^2 + bx + c$ …". On page 315 of *Algebra 2*, the definition of a quadratic function is one "that can be written in the form $f(x) = a(x-h)^2 + k$ …". These definitions can be reconciled although that is never explicitly done. Perhaps no harm is done, but it is certainly idiosyncratic.

Under the heading "identifying quadratic functions", in *Algebra 1*, tables are determined to be quadratic functions (they don't mention the very limited domains) using "second differences" because it is asserted that second differences are constant for all quadratic functions. No justification is suggested.

Later, page 598 of *Algebra 1*, it is asserted, without justification, that "every graph of a quadratic function … is symmetric about a vertical line through its vertex called the axis of symmetry." This line of symmetry is very important and a formula is given for it, but again, with no justification.

On page 652 of *Algebra 1,* the quadratic formula is nicely derived by using the technique of completing the square.

Our real work begins in Chapter 5 of *Algebra 2*. We have already mentioned the definition of a quadratic function as a quadratic in the vertex form. The book then emphasizes, page 318, the transformations that take the function x^2 to the vertex form. The symmetry about the y-axis of x^2 is determined on page 323 and it is asserted, "this shows that parabolas are symmetric curves". This is perfunctory. Because of the importance of symmetry, it would be nice to see this carried through the various transformations between x^2 and the vertex form a little more explicitly.

With quadratic functions written in the vertex form, max/min problems are direct and easy as the solution is given from knowledge of where the vertex is.

The standard form is introduced on the next page, page 324, and the axis of symmetry and vertex are computed by showing how the vertex form is related to the standard form. This is done algebraically for the general case, and this is the only program that does this.

With the formulas at hand, min/max problems for quadratics are dealt with starting on page 326. The problems are straightforward since they involve plugging into formulas that have been derived. To make the problems more complicated, numbers that require the use of a calculator are often used.

There are a number of good, purely algebraic exercises for finding max/min and the axis of symmetry. There are also a number of good word problems but most have a quadratic function stated in the problem. On page 329, problem 32 is a very nice word problem that requires the student to set up the quadratic function and find the value for x that maximizes the function. If symmetry is used, it is unnecessary to use the formulas to solve the problem.

It would be desirable to be able to use the roots of a quadratic function and symmetry to find the line of symmetry. On page 333 this is accomplished with a picture and the statement: "These zeros are always symmetric about the axis of symmetry." More time spent explaining this would be useful.

Section 5-4 is about completing the square, an important mathematical technique that is used often in college level mathematics courses. Completing the square is how the quadratic formula is derived to solve quadratic equations. At a more sophisticated level, by using the technique of completing the square on a quadratic function, the representation changes from the standard form into the vertex form. This is demonstrated by examples and exercises in Section 5-4 allowing students both to solve equations and to put quadratic functions in vertex form (and thus find the symmetry and max/min) without the necessity to memorize formulas. Problem 85 on page 348 is another problem that requires the student to find the function and then find the maximum.

There are lots of good problems.

Summary: *Algebra 2* is much better than *Algebra 1*. In *Algebra 1* formulas are given to you and the unjustified "second differences" is used. One could think of this as an empirical introduction, but it is perhaps better to avoid such an approach. The attempt to deal with symmetry is begun in *Algebra 2* by actually showing that x^2 is symmetrical around the y-axis. Transformations that take this parent function to the general quadratic in vertex form are quite explicit. Then *Algebra 2* shows how to go between the general standard and vertex forms of quadratics. This is the best attempt at doing symmetry of the four programs. Much more detail would be better even here. The vertex and the line of symmetry are actually calculated. Although there are many well-conceived problems, there are few that require the students to produce quadratic functions or equations rather than solve those given in the problem.

Max/min problems for quadratics in *Discovering Algebra*

Quadratic functions are dealt with in two books, *Algebra* and *Advanced Algebra*. An overview of their approach is relevant before our review begins. Quadratic functions and the algebra of quadratic functions seem to be of less interest than graphs in *Discovering Algebra*. More precisely, the objects of study are pictures of graphs that we are usually told are parabolas. The functions that give these sets of points as graphs are determined and used to study the graphs further. Quadratic functions are analyzed but there are issues with the coherence of the development, which are discussed below.

There is a lot of repetition in the two books. In particular, *Advanced Algebra* does not assume the coverage of quadratic functions in *Algebra*.

We start with *Algebra* on page 425. Here we are told that the graph of $y = x^2$ is a parabola and symmetry is determined through following the directive and question in Step 8: "Draw a vertical line through the point (0,0). How is this line like a mirror?" A proof of symmetry is not given in the book but students are given the opportunity to discover the proof. On page 447 we are given the image of a transformation of $y = x^2$ and are asked to find the transformation, i.e. the new function that gives the new graph, which is assumed to be a parabola. Mathematically, this is quite discomforting because, unlike an equation, a picture of a graph is not precise mathematical information.

The use of pictures of graphs is common in *Discovering Algebra*. This approach is problematic in the absence of the text providing a multitude of qualifiers.

On page 466 we are given another picture of a graph and asked to find the function "shown in this graph." We are told the graph is a parabola in the solution, something that should have been done in the statement of the problem. The solution requires a more elaborate transformation from $y = x^2$ because the first problem only used translations and this problem also requires some stretching. It is important to note that *parabola* is undefined, so these pictures of graphs are just examples of whatever it is that parabolas are. In particular, even after having been told this last graph represents a parabola, we have no reason to believe it is a transformation of the function $y = x^2$. Unspoken assumptions are necessary all along the way for the text to be mathematically precise.

The bulk of the quadratic material is in Chapter 9 of *Algebra*. We are told that the graph of the height of objects under the influence of gravity is a parabola. We are given the function describing such behavior, a quadratic function in standard form. We are also told that the transformations of $y = x^2$ are quadratic functions. Although it isn't done, it would be quite easy to show symmetry for these transformed functions. This would suffice for the vertex form of quadratics, but the text works with the standard form and assumes symmetry for it as well. Even if the authors did the easy symmetry for the vertex form, they would still not have derived what they use about the standard form. This is a significant flaw.

Although quadratic inequalities are not explicitly developed, there are problems for them, such as 5(d) on page 500.

On page 502 there is a very nice problem that seems to be solved (we are not given the solution) by graphing and probably assuming symmetry. On page 503 the roots of an equation in standard form are found using a calculator. A more elaborate example is worked on page 504. A simple quadratic function is given in standard form. A calculator is used to find approximate roots to 3 decimal places. Symmetry is assumed and the x-coordinate for the line of symmetry is determined by averaging the two roots. This magically turns out to be 1.5 and the 3 decimal places are not needed. If the rounding to 3 decimal places had left us with 1.499 or 1.501, this exercise would have turned out differently. The vertex is found and the vertex form of the equation is determined up to the value of "a" in the vertex form. The first equation and the vertex form are both graphed (using 1 for "a" in the vertex form) on a graphing calculator. The graphs overlap. This overlap determines that the value of "a" is 1. There are so many mathematical fallacies associated with this sequence of steps it is difficult to know where to start and when to quit. We will just point out that the initial equation is $y = x^2 + 3x - 5$ and the discovered vertex form is $y = (x + 1.5)^2 - 7.25$. To determine that they are the same because they look the same on a graphing calculator and in tables evades and subverts the real goals of both the standards and algebra. To show that these functions are the same by using algebra is a simple matter of expanding the vertex form. Graphing calculators are repeatedly used to graph quadratics and solve for roots and the vertex.

By page 510 we learn how to rewrite specific vertex form quadratics in general form. This is not done for the general case, but we are shown how to do it and practice is given. It is also easy to do this for quadratics written in factored form. We know we cannot always factor a quadratic in general form, but in order to see symmetry and to find max/min (i.e. the vertex), we must be able to rewrite the general form of a quadratic in vertex form. This has not been done yet, but the symmetry has been assumed. Because we do not yet know how to write standard form quadratics in vertex form, the book continues to use graphs to determine the vertex and find the vertex form. This approach to the mathematics is worrisome because the text wrongly assumes the vertex form can legitimately be found from a graph and teaches students that this approach is an acceptable way to solve this type of algebra problem.

At least one problem like this on page 512 appropriately expects students to check their answer by expanding the vertex form rather than graphing.

There are some nice sidelines in *Algebra*. We identify two of them. First, on page 507, the authors point out that parabolas are conic sections. There is no justification, but it is a good thing to know. Second, on page 524 there is a definition of parabolas in terms of a focus and a directrix. Although this is not connected to quadratics in *Algebra*, in Chapter 9 of *Advanced Algebra*, this definition is shown to be equivalent to some quadratic equations. This is a nice, but not required, sideline.

In Lesson 9.6 of *Algebra*, starting on page 525, we are taught how to complete the square. An example is given for functions as well as for solving equations. Technically, although it is not done for the general case, we can manipulate quadratics in standard form to be in the vertex form, which is an important foundation for all work with max/min problems. Converting between the various forms for a quadratic function has now been done. Completing the square is then used to find the quadratic formula.

We now review *Advanced Algebra.* On page 194 we are told that $y = x^2$ is symmetric without benefit of explanation. Transformations of this "parent" function are done. Given the importance of symmetry, it would be easy here and in the vertex form to prove symmetry. A little time spent developing a proof of symmetry would be nice. The false logic used goes something like this: $y = x^2$ is symmetric and it is a parabola; since we are told quadratic functions are parabolas, they must also all be symmetric.

In many ways the development of quadratic functions is better in *Algebra* than in *Advanced Algebra*. At the beginning of the study of quadratic functions in *Advanced Algebra*, in Chapter 7, page 360, we are introduced to the "finite differences method" and, without providing justification, quadratic equations are produced from functions with constant second differences. The fact that quadratic functions produce second differences, something not that hard to do, is not demonstrated. An example or two is all the text offers.

On page 368 the text states explicitly: "Recall from Chapter 4 that every quadratic function can be considered as a transformation of the graph of the parent function $y = x^2$." This is incorrect. The standard form was not derived. The vertex form of quadratic functions was derived in this way, but without connecting the vertex form to the standard form we do not know if **all** quadratics in standard form can be constructed in this way. These unproven statements about symmetry are used throughout the text.

Finally, completing the square for the standard form is left as an exercise on page 380. Students are told to derive formulas for the vertex from this activity. Then these formulas are used to deduce the quadratic formula. The proof of the quadratic formula is not as clear as in *Algebra*.

Summary: In addition to the failure to deal with basic foundational issues associated with symmetry, the material is skewed toward the study of graphs rather than the study of quadratic functions. The low status afforded the functions and the algebra of the functions is disturbing. While the two textbooks have a nice collection of problems, including max/min problems, they have seldom done the mathematics to justify the solutions they find. *Algebra* tends to be better than *Advanced Algebra* in its development of the mathematics. However, logic breaks down at many points in the presentation. We have issues with the lack of development of symmetry, and going through three decimal place approximations using a calculator to end up with a precise function makes no sense. In this way and others, graphing calculators are used to undermine the structure of the mathematics. The logic of finite differences is not presented. *Discovering* relies on its claim to have shown that all quadratics are transformations of the function x-squared but it is a spurious claim. These problems detract from this program and keep it from being consistent, coherent, and mathematically sound.

Max/min problems for quadratics in *Core-Plus Mathematics*

Quadratics are spread throughout *Courses 1*, 2 and 3, with most of the material in *Course 1*, Unit 7, and just some finishing touches in *Courses* 2 and 3.

We begin our review on page 474 of *Course 1,* where, in Problem 3, symmetry and max/min are studied for the simple quadratic function $y = ax^2$. This is followed immediately by studying the same properties when you add a constant to get $y = ax^2 + c$. This is all taught through exercises, but it is done well enough and the Teacher's Guide clears up any problems.

Next, *Course 1* moves on to quadratic functions of the form $y = ax^2 + bx$. This is a nice approach since this function always factors as x(ax+b) and has the obvious roots at x=0 and x=-b/a. The line of symmetry has to be halfway between these two roots and so is at x=-b/2a. The max/min can then be calculated and none of this changes if you add a constant to get the general form for a quadratic function, $ax^2 + bx + c$.

There is a serious problem with what the authors have done to determine the line of symmetry. It was straightforward for them to show symmetry for $y = ax^2 + c$. To calculate where the line of symmetry is for $y = ax^2 + bx$, they **assume** symmetry and then use the two roots (the line of symmetry must be halfway between). Symmetry for $y = ax^2 + bx$ is not obvious, but can be proven by plugging in x=-b/2a + z and x=-b/2a –z to see it (you get the same answer for both). Proof of symmetry for $y = ax^2 + bx$ didn't happen, perhaps because the program tends to shy away from algebraic manipulations, but more likely because the authors didn't realize it had to be done.

There are lots of nice problems, in particular of the max/min type that we are interested in. However, the justification for what the text does is missing. The lack of justification is related to the summary of the mathematics on page 478, part c:

> How are the graphs of functions defined by rules like $y = ax^2 + bx$ $(b \neq 0)$ different from those of functions with rules like $y = ax^2$? What does the value of *b* tell about the graph?

These are highly technically questions, and the answers in the Teacher's Guide are technical as well. Unfortunately, the answers aren't explained or justified.

Although not directly part of this review, question 14(a) on page 484 is extremely difficult:

> Describe how you could position a plane intersecting a cone so that the cross section is a parabola.

The Teacher's Guide just gives the answer: "Position the plane so that it passes through the cone parallel to one edge of the cone." There is no indication that this is a mathematics question of significant depth. Parabolas are defined as graphs of quadratic equations. The connection to conic sections is non-trivial but not investigated as mathematics. No justification for the answer is given.

Starting on page 491 there are some pages that show different forms of the same quadratic expression. Computer algebra systems are brought in quickly. The algebraic skills developed here are minimal.

The quadratic formula is brought in on page 515, but the authors make clear that its proof is put off until *Course 3.* This shows that *Core-Plus* sometimes knows when things have to be proven, suggesting that when they didn't properly develop symmetry, they were completely unaware of the problem.

Not much is done with quadratics in *Course 2*, but what is done assumes the unproven symmetry. On page 355 there is a perfectly nice derivation of the quadratic formula.

Lesson 2 of Unit 5 of *Course 3* wraps up quadratic functions. The first Investigation explores the vertex form of a quadratic function. The investigation finds the max/min and the intercepts. There is no mention of symmetry. Symmetry of the graph is easy to see when a quadratic function is written in vertex form, and so is the max/min. Since symmetry is such an important property of a quadratic function, it is most important to learn how to rewrite a standard form quadratic function in vertex form so you can see the max/min and the symmetry. This is more sophisticated process than merely using the same technique on a quadratic equation. *Core-Plus* nicely sidesteps this difference by teaching completing the square for quadratic expressions. The text then applies completing the square both to equations to produce the quadratic formula, and to functions to obtain the vertex form. This is nicely done.

On pages 112-124, numerous good problems with quadratic inequalities are found.

Summary: There is a lot to like in how *Core-Plus* develops quadratic equations and functions, especially with respect to our focus on max/min problems. There are lots of such problems. There are two serious problems with the soundness of the mathematics. First, there is the failure to take care of the foundation and show symmetry. Second, there is the minimizing of the importance of algebraic skills.

Max/min problems for quadratics in *Glencoe McGraw-Hill*

Preliminary to quadratic functions and equations in *Algebra 1*, Chapter 7 develops skills with the arithmetic of polynomials and Chapter 8 does factoring. With solid manipulative skills to rely on, quadratics are made much easier to deal with. There is a rocky start to quadratics in Chapter 8, page 468, with a picture of the famous Gateway Arch in St. Louis, and the statement: "Quadratic equations can be used to model the shape of architectural structures such as the tallest memorial in the United States, the Gateway Arch in St. Louis, Missouri." This would be a great observation, except that if the Gateway Arch were a parabola it would probably fall down. It is a catenary, not a parabola.

The problems throughout *Glencoe* are excellent, including even problems in the chapter on factoring. For example, on page 497, problem 41:

> A square has an area of $9x^2 + 30xy + 25y^2$ square inches. What is the perimeter of the square? Explain.

This is far more than a simple factoring exercise.

Chapter 9, page 525, begins the study of quadratic functions. There are none of the hidden assumptions that are common in the other programs under review. Instead, we get blatant, up-front, assumptions. We are handed formulas and facts for all the important properties that algebra is designed to compute. There is no effort to explain how the formulas are derived. Much of the rest of quadratics is just a matter of plugging into the unproven formulas. On page 525 we are given the definition of a quadratic function as a quadratic in the standard form. We are told that the graph is a parabola and that "parabolas are symmetric about a central line called the axis of symmetry". Then we are given a formula for the x coordinate of the axis of symmetry, -b/2a. We are now told where the max/min will happen, i.e. the vertex (with x coordinate –b/2a).

While other programs subtly assume symmetry and work from there, this program goes much further than that. Every formula that requires derivation is just handed to us. The entire chapter on quadratics is undermined by the blind use of formulas with no explanation.

Lesson 9-3 is about transformations of quadratic functions and one could hope that they would justify some of the stated results that the program has used so far, but this is not the case. The text does not deal with symmetry and only deals with adding "c" or multiplying by "a", but never deals with the "bx" term or translation to the left or right.

On page 558 the quadratic formula is given. The authors have completed the square on the standard form for individual quadratic equations, but not for the general form. This treatment is inadequate.

The *Algebra 2* book starts from scratch. It begins on page 249 with graphing functions by making tables. Any hope that this second book might be more sophisticated and justify formulas taken for granted in *Algebra 1* is dashed immediately on page 250 where we are told again about symmetry and given the formula for the axis of symmetry and the vertex. Deriving these formulas is one of the driving forces behind algebra at this stage, so the text undermines the necessity for algebra.

In addition to ignoring foundational material, the authors are sloppy in other ways. The motivational problem that opens Lesson 5-1 on graphing quadratic functions reads as follows:

> Eddie is organizing a charity tournament. He plans to charge a $20 entry fee for each of the 80 players. He recently decided to raise the entry fee by $5, and 5 fewer players entered with the increase. He used this information to determine how many fee increases will maximize the money raised.

Although they suggest a quadratic function represents the situation, there is too little information to justify the solution provided on page 253 It turns out that the authors are assuming that a linear increase in entry fee results in a linear decrease in players, information that should be in the statement of the problem.

Despite this example, many of the problems are very nice.

On page 268 the factored form of a quadratic equation is introduced. In this section, FOIL is used as a mnemonic for multiplying two binomials rather than the mathematically correct approach of learning distributivity.

On page 284, completing the square is developed for solving equations. Finally, on page 292, the quadratic formula is derived. The ultimate goal, transforming the standard form of a quadratic function into the vertex form, is done on page 305. No mention of the importance of this procedure for showing symmetry and solving max/min problems is given.

Inequalities for quadratics are done in Lesson 5-8.

Summary: While other programs corrupt the foundation of the study of quadratic functions by assuming symmetry of the graphs, *Glencoe* just assumes everything you want to know. After *Glencoe*'s good programs for linear functions and geometry, its work with quadratics is disappointing.

Summary of max/min problems for quadratics

None of the programs properly develops symmetry, but all use it extensively, mostly without providing any or adequate justification. *Holt* comes the closest to developing symmetry and a case could be made that they have done symmetry completely. Many more details would have been nice. *Glencoe* is the worst in this regard because it makes more unjustified assumptions than the other programs. It reduces much of the work to plugging into formulas that were not derived. *Discovering* is riddled with logical fallacies. *Holt* and *Glencoe* develop algebra skills before attacking quadratics, but *Discovering* and *Core-Plus* shy away from any emphasis on such skills.

The ideal max/min word problems require the student to set up the quadratic function and then find the maximum or minimum. Lots of problems in all the programs give the student the equation. *Holt* only has a couple of max/min word problems that require the setting up of the function. The other three programs all have several such problems.

None of these programs is anywhere near an ideal choice. If one ignores *Algebra 1, Holt* seems to be the best. *Core-Plus* is the least objectionable of the three remaining and *Glencoe* the most objectionable. *Discovering* has serious flaws. The severity and nature of the flaws in *Discovering* and *Glencoe* make their texts unacceptable.

Geometry: triangle sum theorem

We have chosen to evaluate how the three programs present the theorem that the sum of the angles of a triangle is 180 degrees, which falls under the standard:

G.3.A *Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle*.

The theorem is a fundamental theorem of Euclidean geometry and it connects many of the basics in geometry to each other. For example, the theorem depends on a good understanding of parallel lines and lines that cross them and angles associated with them all. This theorem will be looked at carefully, more for the general coherence and logical progression of the geometric material leading up to the theorem than for the theorem itself. The main concerns will be the foundation for understanding both the geometry of the situation and the logic it depends on.

The triangle sum theorem in *Holt*

The triangle sum theorem sits at the top of a pyramid of postulates, definitions, other theorems, and logic, all of which must be in place to make sense of the result.

We find a proof of the "triangle sum theorem", i.e. the theorem of Euclidean geometry that the sum of the angles is 180 degrees, on page 223 of *Geometry*. The initial step of the proof requires the "parallel postulate" from page 163. This is Playfair's version of Euclid's fifth postulate that postulates the existence of a unique parallel line through a point not on the first line.

The next required result is the "alternate interior angles" theorem from page 156. It, in turn, requires the "corresponding angles postulate" from page 155 and the "vertical angles" theorem from page 120. Its proof requires the "linear pair" theorem from page 110 and the "congruent supplements" theorem from page 111.

At various places along the way we also need postulates about addition and substitution from page 104.

At the base of this pyramid are a number of definitions: of parallel lines, alternate interior angles, straight angles, vertical angles, corresponding angles, a transversal, congruent, supplementary and linear pairs, to name some of the obvious. The definitions all seem to be there and are nicely done.

One flaw is the insertion of redundant postulates. To take only one example, on page 162 we are given the Converse of the Corresponding Angles Postulate. This can be proven from the parallel postulate and the corresponding angles postulate.

Summary: The main criticism of this program is the use of redundant postulates. The mathematics is all in order. This is a sound, coherent presentation of the triangle sum theorem.

The triangle sum theorem in *Discovering*

The title of the final chapter, Chapter 13, *Geometry as a Mathematical System*, captures the main problem with *Discovering Geometry*. The first 690 pages are not geometry as mathematics; the mathematics part of this textbook begins on page 691. This seems a little late. The text points out (page 692), since 600 B.C.E. "Mathematicians began to use logical reasoning to deduce mathematical ideas." Postulates are mentioned for the first time on this page as well. Further explanation for what has gone on in the first 690 pages is given on page 693:

> You used informal proofs to explain why a conjecture was true. However, you did not prove every conjecture. In fact, you sometimes made critical assumptions or relied on unproved conjectures in your proofs.
>
> In this chapter you will look at geometry as Euclid did. You will start with premises: definitions, properties, and postulates. From these premises you will systematically prove your earlier conjectures. … You will build a logical framework using your most important ideas and conjectures from geometry.

Being "systematic" and building a "logical framework" is essential to mathematical structure, and mathematical structure is mathematical content. Together, they constitute much of the mathematical soundness considered in this review.

The triangle sum theorem is proven on page 706 in a nice clean way. The necessary alternate angles theorem is proven on page 704, as is the vertical angles theorem. On pages 696-697 the postulates of geometry are stated, in particular the ones needed for the triangle sum theorem: the corresponding angles postulate, the linear pair postulate, the angle addition postulate, and the parallel postulate (the last is Playfair's postulate, equivalent to Euclid's fifth postulate, that makes this Euclidean geometry). On page 694 the properties of arithmetic that are used are given. The discussion and development are very sparse, but 690 pages of discussion precede this chapter.

From beginning to end we have a nice, coherent, concise, proof of the triangle sum theorem from the beginnings of geometry on page 691 to the final proof on page 706. Unfortunately, the definitions of all the terms used, and there are many, are presumed to be somewhere in the first 690 pages of the book, so our review requires us to investigate this part of the book.

A proof of the triangle sum *conjecture* shows up on page 202. This is where mathematical structure and logic break down and confusion dominates. On this page we have what appears to be a nice clean proof of the triangle sum theorem, here called the triangle sum conjecture. Even after the proof it remains the triangle sum conjecture for the reasons explained later on page 693 quoted above and again here:

> … you sometimes made critical assumptions or relied on unproved conjectures in your proofs.

In particular, the proof uses the linear pair conjecture and the alternate interior angles conjecture. The foundation is, rather obviously, lacking. More than that is missing, though. The statement of the triangle sum conjecture reads as follows:

> The sum of the measures of the angles in every triangle is ____?____.

The blank is to be filled in by the student either from an investigation or by reverse engineering the proof, which is quite explicit about what the conjecture should be.

On page 129 we find two of the necessary ingredients of the proof of the triangle sum conjecture:

> The corresponding angles conjecture: If two parallel lines are cut by a transversal, then corresponding angles are ___?___ .

> Alternate interior angles conjecture: If two parallel lines are cut by a transversal, then alternate interior angles are ___?___ .

These are not stated adequately. On the previous page we find some of the definitions we were looking for. There is a diagram of two parallel lines and a transversal with all the angles labeled. Transversal is defined, but the definition of corresponding angles is stated as:

> One pair of **corresponding angles** is ∠1 and ∠5. Can you find three more pairs of corresponding angles?

A similar "definition" is given for alternate interior angles. These, of course, do not qualify as mathematical definitions. Discovery has its place. Discovering theorems is nice, but definitions cannot be discovered. This discovery disrupts the foundation of the mathematics which chapter 13 is supposed to provide.

Continuing our trek backwards through the book we find, on pages 122-123, the linear pair conjecture and the vertical angles conjecture stated as follows:

> If two angles form a linear pair, then ___?___ .

> If two angles are vertical angles, then ___?___ .

From a mathematical point of view, these are not stated clearly.

Continuing, we find a real definition of parallel lines on page 48. On page 50 we find many of the definitions we have been looking for: supplementary angles, vertical angles, and linear pair of angles. The definitions are not stated in English sentence form but with pictures and identification of angles that meet the definition. It is not completely satisfactory.

Finally, the triangle sum theorem depends on Euclidean geometry. It is Euclid's fifth postulate that distinguishes Euclidean geometry from non-Euclidean geometry. Euclid's rather awkward fifth postulate is frequently replaced in the text with the equivalent Playfair's version: Through a point not on a given line, you can construct exactly one line parallel to the given line. This is given to us on page 696 with the other postulates. Preceding the triangle sum conjecture on page 202 we need to find something resembling this in order to use it in the "proof". The only place this exists is on page 163 where such a parallel line is constructed using "patty paper," the mathematical foundations of which are lacking.

Summary: The text consists of 690 pages of inductive geometry followed by a short attempt to do rigorous deductive geometry. Unfortunately, the rigorous attempt depends on vague and "discovered" definitions scattered throughout the first 690 pages. This is a highly unsatisfactory geometry text.

The triangle sum theorem in *Core-Plus Mathematics*

The triangle sum theorem appears as an exercise on page 45, Unit 1, Lesson 2, of *Course 3.* The proof is standard and developed through a series of questions. Our job is to trace back through the definitions, postulates, and theorems used to be sure that all is in place to make this mathematically coherent. Formal geometry involving postulates starts on page 30 so the development should be quick and easy to access given that there is only 15 pages to do it in.

We find definitions for **linear pair** and **vertical angles** on pages 30-31, followed by the **Linear Pair Postulate** that says the sum of the angles for a linear pair is 180 degrees. This is really our definition of degrees, and it gives us that an angle that makes a straight line is 180 degrees. The Linear Pair Postulate isn't said quite properly because knowledge of degrees is really assumed. Next we find the **Vertical Angles Theorem**.

On page 32 we get a definition of **perpendicular** in terms of right angles, but right angles are not defined. Trying to trace back through *Courses* 1 and 2, it seems that right angles are known coming into *Core-Plus* and so we never see a proper definition. This is a non-trivial flaw in the development of the triangle sum theorem.

On page 35 we find definitions of **parallel lines** and **transversal** lines, as well as the names of various related angles. The **Parallel Lines Postulate** shows up on page 38:

> In a plane, two lines cut by a transversal are parallel if and only if corresponding angles have equal measure.

This postulate is the crucial postulate needed for much of the proof of the theorem under consideration. The first step in the proof of the theorem is to construct a line through one vertex of the triangle that is parallel to the opposite side. Various angles are then shown to be equal. This postulate takes care of the equality of these angles. What is left, aside from some facts assumed about arithmetic, is the actual construction of the parallel line.

This construction is normally carried out using Playfair's postulate that says you can always do just that; find exactly one parallel line through a point not on the first line. This is equivalent to Euclid's rather complex fifth postulate, the postulate that defines Euclidean geometry rather than non-Euclidean geometry. There are many sets of equivalent postulates, and Playfair's postulate is an acceptable alternative to Euclid's fifth postulate. However, the postulate used by *Core-Plus* is quite different and requires a different proof of the triangle sum theorem.

The "Parallel Lines Postulate" does tell us that if there is a parallel line through an external point, it is unique although that is not proven in the text at this stage. This information is important, but we also need a proof of the existence of a parallel line through the point. The way this is done is by a construction shown to us on pages 33-34. First, drop a perpendicular from the point to the line, and then construct a perpendicular at that point to the new line. Now apply the postulate. All the angles are (undefined) right angles and so the last line and the first line are parallel. The problems with this proof when evaluated in terms of logical development and mathematically coherence are overwhelming. The justification of the construction involves (bottom of page 33) showing two triangles are congruent. Triangle congruency is studied in *Course 1* starting on 371, but this is done without benefit of foundation (definitions, postulates, etc). Nonetheless, triangle congruency is used here. The formal study of triangle congruency starts in *Course 3* on page 195. Using pre-postulate informally proven theorems or statements to prove results formally, only to have those informal theorems show up much later in the formal geometry is not mathematically sound. It is called circular reasoning.

Summary: The program fails to build geometry up from foundations in a mathematically sound and coherent way. Informal geometry is studied before formal geometry and "theorems" from informal geometry are used to prove theorems in formal geometry destroying any logical coherence in the program. One significant goal of a geometry course is to teach logic, and this program fails on that account. It is quite an unsatisfactory geometry program.

The triangle sum theorem in *Glencoe McGraw-Hill*

The theorem appears on page 244 of the geometry text. The proof is straightforward. It is detailed and requires substitution, covered on page 134, the angle addition postulate, page 149, linear pair, page 46, supplementary angles, page 47, and various other angles, page 172. Also necessary is the alternate interior angles theorem, found on page 179. This depends on the vertical angle theorem of page 152, left as an exercise and the supplement theorem on page 150, also left as an exercise.

What remains is the drawing of the auxiliary line through one vertex parallel to the opposite side. This is accomplished by the parallel postulate on page 206. This postulate is Playfair's version of Euclid's fifth postulate and it gives us exactly what is needed, a parallel line through a point not on the line.

There is a bit of overkill at work in here. On page 205 we have a "converse of corresponding angles postulate", but it would follow as a theorem from the parallel postulate and the corresponding angles postulate. Better to have too many postulates than too few, or, to have them in the wrong order than not to have them at all.

Summary: This program provides a logical development of the material. Along with the formal geometry, some informal geometry is also used. For example, the text shows how to bisect an angle using a compass and a straightedge on page 40, but only justifies it as a theorem later on page 325. This is acceptable because they do not use their informal geometry in the formal proofs. This is a sound proof of the triangle sum theorem.

Summary of the triangle sum theorem

Holt and *Glencoe* are straightforward, logical, coherent, and sound geometry programs.

Neither *Discovering* nor *Core-Plus* is acceptable as a geometry program. *Discovering* puts off rigorous geometry until the end of the book, and then relies on vague definitions from the previous part of the book. This program must rank higher than *Core-Plus*, which uses informal theorems to prove formal theorems that are then used to prove the informal theorems formally. This is circular reasoning.

Summary of all evaluated categories in all programs

Geometry is important, so the unacceptable nature of geometry in *Discovering* and *Core-Plus* makes these programs unacceptable. The flaws in these geometry programs are such that they could not easily be compensated for by a teacher, even with the help of supplementation. Both *Holt* and *Glencoe* have acceptable proofs of the triangle sum theorem. *Holt = Glencoe > Discovering > Core-Plus*.

The core of algebra developed with linear and quadratic equations and functions is essential for students who plan to move forward with mathematics. All of the programs are seriously flawed when it comes to graphing linear equations and the teacher should redress this inadequacy through supplementation. Two of the programs, *Discovering* and *Core-Plus*, downplay algebraic structure and skills so heavily that the linear equation/function part of their programs is unacceptable. Both *Holt* and *Glencoe* are much better than *Discovering* and *Core-Plus* for linear equations and functions. *Glencoe > Holt > Discovering > Core-Plus.*

The foundation for the study of max/min problems in quadratics rests on the development of symmetry. Only *Holt* comes close to doing this well. There is much to like about *Core-Plus*. *Discovering* has serious flaws. Its goals are not those of algebra. *Glencoe* ignores the power of algebra to derive important information and just gives it to the student. Both *Discovering* and *Glencoe* are unacceptable. *Holt > Core-Plus > Discovering > Glencoe*.

Discovering is unacceptable in all three categories studied. *Core-Plus* is unacceptable in two out of three categories. *Glencoe* is unacceptable in one category. *Holt* is the only program acceptable in all three categories, but it cannot be recommended with enthusiasm.

There is a very disturbing aspect to the failure to do the foundational work with the graphing of linear functions and the symmetry for quadratic functions. These are long books, in some cases over 1,000 pages. The missing foundational work rarely requires more than a couple of lines, yet these important concepts do not make it into the texts.

Keep in mind that these evaluations are for three specific issues in each program, they are not evaluations of the total program and should not be read that way. However, how these major topics in high school mathematics are handled should say a lot about each program.