Exam #1, October 30, Calculus I, Fall, 2006, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: ___________________________ Date: ______________

TA Name and section: _____________________

**NO CALCULATORS, NO PAPERS, SHOW WORK.** This exam may well be too long. Use your time wisely. (40 points total)

1) (2 points) Compute: \( \lim_{x \to 0} \frac{\sqrt{2-x} - \sqrt{2}}{2x} \)

2) (2 points) Compute: \( \lim_{x \to \infty} \frac{e^{-x}}{1-e^{-x}} \)
(3) (2 points) Compute: \( \lim_{x \to 0} \frac{\sin(x) \cos(x)}{2(1-x)} \)

(4) (2 points) Compute: \( \lim_{x \to 0} \frac{(e^x - 1)^2}{\sin^2(x)} \)
(5) (2 points) What is the equation for the tangent line to \( y = x^2 \) at \( x = 2 \)?

(6) (2 points) Let \( y = f(x) \). What is \( y' = \frac{dy}{dx} \)?
(7) (2 points) Let \( y = \sqrt{1 + f(x)^2} \). What is \( y' = \frac{dy}{dx} \)?

(8) (2 points) Let \( y = f(x)^y \). What is \( y' = \frac{dy}{dx} \) when \( x = 0 \) and \( y = 3 \) if \( f(0) = 2 \) and \( f'(0) = 4 \)?
(9) (2 points) Let \( y = (1 + \cos(x))^3 \). What is \( y' = \frac{dy}{dx} \) ?

(10) (2 points) Let \( y = 2^{\sqrt{x^2-1}} \). What is \( y' = \frac{dy}{dx} \) ?
(11) (6 points) Sketch $y = f(x) = x(x^2 - 1)$. Label all important properties. Show work.
(12) (6 points) The surface area of a sphere of radius $r$ is $4\pi r^2$ and the volume of the sphere is $\frac{4}{3}\pi r^3$. You have 100 square inches of material to cover a sphere and a cube (with edge $x$). You want the minimal total volume enclosed by the sphere and the cube. What must $r$ and $x$ be? (Good partial credit for the ratio of $r$ to $x$.)
(13) (8 points) Two idiots have been driving around the Cartesian plane. At time $t$, the first idiot is at point $(3t, 4t + 4)$ and the second idiot is at $(t, 2t)$. How fast are they moving towards each other (or away from each other) at time $t = 0$? When are they closest to each other? How close are they and where are they when they are closest?