Exam #1, October 15, Calculus III, Fall, 2008, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: ___________________________ Date: _______________

TA Name and section: _______________________

**NO CALCULATORS, NO PAPERS, SHOW WORK.** (22 points total)

1. (1 point) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be given by $f(x, y, z) = x^3 + y^3 + z^3$. Compute the gradient of $f$.

2. (1 point) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be given by $f(x, y, z) = x^3 + y^3 + z^3$. Compute the Laplacian of $f$. 
3. (1 point) Let \( F : R^3 \rightarrow R^3 \) be given by \( F(x, y, z) = (x^3, y^3, z^3) \). Compute the divergence of \( F \).

4. (1 point) Let \( F : R^3 \rightarrow R^3 \) be given by \( F(x, y, z) = (x^3, y^3, z^3) \). Compute the curl of \( F \).
5. (1 point) Let $c : \mathbb{R} \to \mathbb{R}^3$ be given by $c(t) = (t^3, t^3, t^3)$. Compute the tangent line to $c$ at the point $t = 1$.

6. (1 point) Let $c : \mathbb{R} \to \mathbb{R}^3$ be given by $c(t) = (t^3, t^3, t^3)$. Compute the length of the curve between $t = 0$ and $t = 1$. Do this by setting up and evaluating the integral.
7. (2 points) Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be given by $f(x, y, z) = (x^3 + y^3 + z^3, x + y + z)$. Compute the derivative of $f$ at the point $(1, 1, 1)$. 
8. (2 points) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be given by $f(x, y, z) = x^3 + y^3 + z^3$. Let $x, y,$ and $z$ be functions of $t$ such that $x(0) = y(0) = z(0) = 1$ and $x'(0) = 1$, $y'(0) = 2$, and $z'(0) = 3$. Compute $df/dt$ at $t = 0$. 
9. (2 points) Let \( f : \mathbb{R}^3 \to \mathbb{R} \) be given by \( f(x, y, z) = x^3 + y^3 + z^3 \). Compute the directional derivative of \( f \) in the direction \((1, 1, 1)\) at the point \((1, 1, 1)\).
10. (2 points) Give an equation for the tangent plane of the graph of the function \( f(x, y) = xy \) at the point \((1, 1)\).
11. (2 points) Give an equation for the tangent plane for the surface given by $x^3 + y^3 + z^3 = 3$ at the point $(1, 1, 1)$. 
12. (2 points) Find all points on the surface given by \(x^2 + y^2 - z^2 = 4\) that are closest to the origin. Use Lagrange multipliers.
13. (2 point) Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, $f(x, y) = \sin(x)\cos(y)$. There are an infinite number of critical points. Find the two critical points closest to the origin with $x \geq 0$ and $y \geq 0$. 
14. (2 points) Calculate the second order Taylor approximation for the two critical points in the previous problem and determine whether each is a local maximum, minimum or saddle point.