Exam #2, November 1, Calculus II (107), Fall, 2013, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): ______________________________ Date: _________________

Name (print): _________________________________

TA Name and section: __________________________

NO CALCULATORS, NO PAPERS, SHOW WORK. (46 points total, not counting 2 bonus points)

Write something even if not sure of answer. 1 out of 2 points for anything that might have been on path to correct answer. Must have work to back it up though on exam.

Throughout the exam we will use the function:

\[ f(x, y) = 3x^2 - y - 2y^2 \]
1. (2 points) What is $\frac{\partial f}{\partial x}$ at the point $(1,0)$?

2. (2 points) What is $\frac{\partial f}{\partial y}$ at the point $(1,0)$?

3. (2 points) What is the gradient of $f$, i.e. $\nabla f$?
4. (2 points) Compute the slope of \( f \) in the positive \( x \) direction at the point \((1, 0)\).

5. (2 points) Compute the direction of maximum slope of \( f \) at \((1, 0)\).

6. (2 points) Compute the slope of \( f \) at \((1, 0)\) in the direction it is maximal.
7. (2 points) (Bonus points, possibly hard question, might want to put it off for now) Give a parametric equation for the tangent line to the graph of $f$ (in 3-space) for $(x, y) = (1, 0)$ in the maximal direction. Express your answer with smallest integers, no fractions or radicals.
8. (2 points) Find the equation (in the form $y = mx + b$) for the tangent line to the level curve at $(1, 0)$. 
9. (2 points) Find the equation for the tangent plane to $f$ for the point (1, 0).

10. (2 points) Find the one critical point for $f$.

11. (2 points) What is $f(x, y)$ at the critical point?
12. (2 points) What is the Hessian of $f(x, y)$ at the critical point?

13. (2 points) What kind of critical point is the critical point: local maximum, local minimum, or saddle?
14. (2 points) If \( x = -1 = y \), and \( \frac{dx}{dt} = 1 = \frac{dy}{dt} \), what is \( \frac{df}{dt} \)?

15. (2 points) If \( x = e^t \) and \( y = e^{-t} \), what is \( \frac{df}{dt} \) at the point \((x, y) = (e, 1/e)\)?
16. (2 points) If $A$ is a $2 \times 2$ matrix such that $Au = -u$ and $Av = -2v$ for $u$ and $v$ non-zero vectors, what is the general solution for the system of differential equations: \[
\begin{pmatrix}
\frac{dx}{dt} \\
\frac{dy}{dt}
\end{pmatrix} = A \begin{pmatrix}
x \\
y
\end{pmatrix}.
\]

17. (2 points) What kind of equilibrium point is $(0, 0)$ in the previous problem?
18. (2 points) In this graph of the direction field for a system of differential equations, what kind of equilibrium point is (0, 0)?
19. (2 points) In this graph of the direction field for a system of differential equations, what kind of equilibrium point is $(0, 0)$?
20. (10 points total) Consider the function $f$ on the domain $x^2 + y^2 \leq 1$. What is the minimum value $f$ takes (2 points) and what point does it take this value (2 points)? What is the maximum value $f$ takes (2 points) and on what two points does it take this value (4 points, 2 each). (Can use Lagrange multiplier or reduce to Calculus I a couple of different ways.)
More space for problem.