1. Determine if the following improper integral converges or diverges. If the integral is convergent compute its value.

\[ \int_{0}^{\infty} xe^{-x} \, dx. \]

2. Let

\[ f(x, y) = \begin{cases} \frac{3x^2y^2}{x+y}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases} \]

(a) Does the \( \lim_{(x,y) \to (0,0)} f(x,y) \) exist?

(b) Is \( f(x,y) \) continuous at \( (0,0) \)?

3. Solve the following first order separable initial value problem

\[ \frac{dy}{dx} = (y-1)(y-2) \]

with \( y(0) = 0 \).

4. Consider the system of linear equations

\[
\begin{align*}
    x_1 - x_2 &= 0 \\
    3x_1 + x_2 - x_3 &= 11 \\
    2x_1 + x_2 + 2x_3 &= 11
\end{align*}
\]

Find the augmented matrix of the above system and use it to solve the system.

5. Consider \( f(x,y) = 3xy - x^3 - y^3 \).

(a) Locate all critical points of \( f(x,y) \).

(b) Classify the critical points of \( f(x,y) \) (i.e., determine if they are local maximum/local minimum or saddle point).

(c) Does \( f \) have a global maximum or minimum on \( \mathbb{R}^2 \)? Briefly explain!

6. Consider the following system of differential equations

\[
\begin{bmatrix}
    \frac{dx_1}{dt} \\
    \frac{dx_2}{dt}
\end{bmatrix} =
\begin{bmatrix}
    -5 & -2 \\
    6 & 3
\end{bmatrix}
\begin{bmatrix}
    x_1(t) \\
    x_2(t)
\end{bmatrix}
\]

Solve the following initial value problem with \( x_1(0) = 5 \) and \( x_2(0) = 3 \).

7. Find the absolute maxima and minima of \( f(x,y) = x^2 + y^2 - 2x + 4 \) on the disk \( D = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4 \} \).

8. Let \( f(x,y) = \sqrt{4x^2 + y^2} \) be a function of two variables.

(a) Compute the directional derivative of the function \( g(x,y) \) at the point \((-2,4)\) in the direction of \( \mathbf{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \).

(b) Find the angle between the vectors \( \nabla f(-2,4) \) and \( \mathbf{v} \).

9. Suppose you wish to enclose a rectangle plot. You have 1600 ft of fencing. Using the material, what are the dimensions of the plot that will have the largest area?
10. Suppose that
\[ \frac{dy}{dx} = y(2 - y). \]
(a) Find the equilibria of this differential equation.
(b) Compute the eigenvalues associated with each equilibrium and discuss the stability of the equilibria.

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