1. Let $f(x, y) = e^{-xy}$ with constraint function

$$x^2 + 4y^2 = 1.$$ 

Using Lagrange multipliers to find all extrema.

2. Consider the system of linear equations

$$
\begin{align*}
3x + 5y - z &= 10 \\
2x - y + 3z &= 9 \\
4x + 2y - 3z &= -1
\end{align*}
$$

Find the augmented matrix of the above system and use it to solve the system.

3. Assume that $f(x, y) = \begin{cases} 0, & \text{if } xy \neq 0, \\ 1, & \text{if } xy = 0. \end{cases}$

(a) Show that $f_x(0, 0)$ and $f_y(0, 0)$ exist.

(b) $f(x, y)$ is not differentiable at $(0, 0)$.

4. Compute

$$\int_0^1 \frac{dx}{(x - 1)^{2/3}}.$$ 

5. Find the absolute maxima and minima of $f(x, y) = x^2 + y^2 + x + 2y$ on the disk $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4\}$.

6. Use the partial-fraction method to solve

$$\frac{dy}{dx} = 2y(3 - y)$$

with $y(1) = 5$.

7. Find the local extrema of

$$f(x, y) = 2x^2 - xy + y^4, \quad (x, y) \in \mathbb{R}^2.$$ 

8. Let

$$A = \begin{bmatrix} 2 & 4 \\ -2 & -3 \end{bmatrix}$$

Without explicitly computing the eigenvalues of $A$, decide whether the real parts of both eigenvalues are negative.

9. Solve the given initial-value problem

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

with $x_1(0) = -3$ and $x_2(0) = 1$.

10. Suppose that

$$\frac{dy}{dx} = y(2 - y)(y - 3).$$

(a) Find the equilibria of this differential equation.
(b) Compute the eigenvalues associated with each equilibrium and discuss the stability of the equilibria.

Department of Mathematics, Johns Hopkins University, 3400 N Charles Street, Baltimore, MD 21218, USA

E-mail address: yli@math.jhu.edu