In agreeing to take this exam, you are implicitly agreeing to act with fairness and honesty.

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1. (10 points) Consider a $2 \times 2$ matrix

$$A = \begin{bmatrix} 4 & 5 \\ -1 & -2 \end{bmatrix}$$

(a) Compute $\text{tr}(A)$ and $\det(A)$.
(b) Find the eigenvalues of $A$. 

2. (20 points) Solve the system of linear equations

\[
\begin{align*}
y + x &= 3 \\
z - y &= -1 \\
x + 2z &= 4
\end{align*}
\]
3. (10 points) Compute the improper integral

\[ \int_{1}^{e^4} \frac{dx}{x \sqrt{\ln x}}. \]
4. **(20 points)** Consider a $2 \times 2$ matrix

$$A = \begin{bmatrix} 3 & -2 \\ 7 & -5 \end{bmatrix}$$

(a) Find the inverse matrix $A^{-1}$ and its determinant $\det(A^{-1})$.
(b) Define $a$ to be the number $\det(A^{-1})$. Suppose that the volume $V(t)$ of a cell at time $t$ changes according to

$$\frac{dV}{dt} = \sin(at) \quad \text{with } V(0) = 3.$$ 

Find $V(t)$. 
5. (20 points) Consider three vectors
\[ x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad y = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad z = \begin{bmatrix} -1 \\ 4 \end{bmatrix}. \]

(a) Compute the dot products \( a := x \cdot y \) and \( b := x \cdot z \).
(b) Let \( P = (1, -1) \) be the point in \( \mathbb{R}^2 \) corresponding to the vector \( x \). Find the line that passes through this point \( P \) and is perpendicular to the vector \( n = \begin{bmatrix} a \\ b \end{bmatrix} \)

where \( a, b \) are defined in (a).
6. (20 points) (a) Solve the differential equation

\[ \frac{dy}{dx} = (y - 2)(y + 1), \quad y(0) = 3. \]

(b) Find the equilibria of the above differential equation and discuss the stability of each equilibrium.