Questions

1. This question concerns the function

   \[ f(x, y) = 1 - yx^2. \]

   (a) Draw the \( y = 1 \) cross-section of \( f(x, y) \).

   (b) Calculate the partial derivative of \( f \) with respect to \( x \) and evaluate it at the point \((2, 1)\).

   (c) On your graph from (a), draw a tangent line to the cross-section whose slope is represented by your answer from (b).

2. (a) Draw a graph showing the \( c \)-level curves of the function \( f(x, y) = x^2 - y \) for \( c = -2, 0, 2 \).

   (b) Mark on your graph from (a) the direction of the gradient vector \( \nabla f \) at the points \((0, 0), (2, 6) \) and \((-1, -1)\). (Note: you do not have to get the right length for these gradient vectors. It is the direction that is important.)

   (c) What is the directional derivative of \( f(x, y) \) in the direction \((1, 0)\) at the point \((0, 0)\)? (You must give a reason or a calculation.)

3. Find the critical point of the function \( f(x, y) = x^2 + 3y^2 \). Is this a local max, local min or saddle? (Show your work.)

4. Calculate each of the following second-order partial derivatives:

   (a) if \( f(x, y) = e^{x^2y} \), find \( \frac{\partial^2 f}{\partial y^2} \);
(b) if \( f(x, y) = \sin(2x + y) \), find \( \frac{\partial^2 f}{\partial x \partial y} \);

5. The attached graph shows the number field for a function \( f(x, y) \). Use the number field to answer the following questions:

(a) For each of the following quantities, say if you would expect it to be positive, negative or zero:
   i. \( \frac{\partial f}{\partial x}(2, 1) \);
   ii. \( \frac{\partial f}{\partial y}(3, 2) \);
   iii. the directional derivative of \( f \) at (3, 4) in the direction of the vector \( \begin{pmatrix} -1 \\ -1 \end{pmatrix} \)

(b) Draw graphs of the \( x = 2 \) and \( y = 3 \) cross-sections of \( f(x, y) \).

(c) Use your graphs from part (b) to explain why it is plausible that (2, 3) is a critical point of the function \( f(x, y) \).
Number field for $f(x/y)$