Video Guide for Linear Algebra 110.201

Fall 2020

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Week # 1

Text: Chapter 1. 12 videos, 112 minutes.

Overview:

Defines systems of linear equations, puts them in matrix notation, and shows how to solve them using row-reduced echelon form. Along the way introduces the notion of the rank of a matrix and the concept of perpendicular using the dot product.

Videos:

1part01.mp4 (7:20)

A simple system of 2 linear equations in 2 unknowns: solves them 2 different ways, and explains their geometric meaning.

1part02.mp4 (7:27)

Examples of simple systems of 2 linear equations in 2 unknowns with 1 solution, infinite solutions, and no solutions, and explains their geometric meaning.

1part03.mp4 (8:37)

Turns the previous examples into matrices and introduces row reduction to find solutions.

1part04.mp4 (7:53)

Moves on to n equations in m unknowns. Puts this in matrix form and gets a start on row reduction.

1part05.mp4 (16:27)

Works an example with 4 equations and 5 unknowns, translates the problem to a matrix and demonstrates row-reduced echelon form to solve the system.

1part06.mp4 (6:51)

Works a simple system for a real-life problem where only integer solutions are acceptable. A “cultural aside” (i.e. content not required, but video is).

1part07.mp4 (4:49)

Defines the row-reduced echelon form of a matrix, (rref(A)). Introduces the coefficient matrix and the augmented matrix and defines the rank of a matrix. It uses these definitions to discuss when there is only one solution, infinite solutions or no solutions, to a system of linear equations.

1part08.mp4 (7:27)

Reinterprets our old simple systems of 2 linear equations in 2 unknowns as a problem about vectors in Euclidean space.

1part09.mp4 (8:55)

Reinterprets a system of n equations in m unknowns as a problem about vectors in Euclidean m-space. Introduces linear combinations and the notation for systems of linear equations, Ax=b. Then develops some of the properties of this notation.

1part10.mp4 (5:43)

Introduces the dot product and how it shows up in our new notation, Ax=b.

1part11.mp4 (12:19)

Introduces perpendicular using the dot product and reduces some problems about perpendicular vectors to systems of linear equations that we know how to solve.

1part12.mp4 (9:25)

Introduces the standard basis for Euclidean n-space and shows some advantages for using it, for example that Ae_j is the j-th column of A.

Weeks # 2 and 3

Text: Chapter 2. 22 videos, 188 minutes.

Overview:

We define and study linear transformations and associate them with matrices, in particular, we need to know how to compute the matrix for a linear transformation given various ways of describing the linear transformation. Numerous examples are worked to illustrate. In particular, we need the inverse of a linear transformation and its matrix (when they exist). Matrix multiplication is developed to study the composition of linear transformations.

Videos:

2part01.mp4 (7:12)

We define a linear transformation from Euclidean n-space to Euclidean m-space by way of a matrix. We then study the identity matrix from several different perspectives.

2part02.mp4 (7:47)

We relate the matrix of a linear transformation to what it does to the standard basis and connect the matrix of a linear transformation to the matrix for a system of linear equations.

2part03.mp4 (8:15)

We take a look at an example of a matrix that rotates the plane and then study all linear transformation from Euclidean 3-space to 1-space. Our final example is to find the matrix for the cross product, i.e. T(z) = v x z for a fixed v in Euclidean 3-space.

2part04.mp4 (10:42)

We study various examples of linear transformations, including all linear transformation from the reals to the reals (i.e. Euclidean 1-space).

2part05.mp4 (9:06)

We use matrices to study the properties of linear transformations and then use those properties to give a new definition of a linear transformation and show that these two definitions are equivalent.

2part06.mp4 (9:46)

We compute the matrices for arbitrary rotations in the plane and one called the "shear."

2part07.mp4 (2:29)

A short video that revisits and improves on the matrix for a shear from the previous lecture.

2part08.mp4 (11:24)

We look at the linear transformation obtained by dropping perpendiculars to a line.

2part09.mp4 (12:38)

In this video we work a concrete problem where we have a line in 3-space and we compute all linear transformation from 3-space to 3-space where the image is perpendicular to the line. In particular, we compute the projection dropping perpendiculars (used to solve a special case) as in the previous video for 2-space.

2part10.mp4 (3:12)

A short video on reflection and dilation.

2part11.mp4 (9:59)

We define, discuss, and prove the properties of the inverse of a linear transformation coming from a matrix.

2part12.mp4 (10:25)

More properties of the inverse and how to compute the matrix for the inverse.

THIS IS A GOOD POINT TO BREAK BETWEEN WEEK # 2 AND WEEK # 3.

2part13.mp4 (8:00)

We compute the matrix for the inverse of an arbitrary invertible linear transformation from 2-space to 2-space.

2part14.mp4 (6:30)

We work some simple examples of problems related to invertible matrices.

2part15.mp4 (15:03)

The composition of linear transformations coming from matrices is shown to be a linear transformation, and so, it must have a matrix. We compute the matrix from the two given matrices, and this defines and justifies the formulas for the multiplication of matrices.

2part16.mp4 (8:36)

Gives a different perspective on the product of matrices that makes it easier to compute.

2part17.mp4 (7:15)

A short excursion into an application of the multiplication of matrices: the chain rule for multivariable calculus (a cultural aside). Then more properties of matrix multiplication are developed (associativity, distributivity, etc.).

2part18.mp4 (7:32)

An example of a linear transformation given in an unusual way where we compute the matrix using our basic principles.

2part19.mp4 (9:54)

A problem involving matrix multiplication that is reduced, like all things in this course, to solving a system of linear equations.

2part20.mp4 (4:57)

Same as above, different problem.

2part21.mp4 (10:18)

Same as above, different problem.

2part22.mp4 (7:01)

Yet another, this time the composition of rotations is shown to give the correct matrix.

Weeks # 4 and 5

Text: Chapter 3. 23 videos, 200 minutes.

Overview:

Subspaces of Euclidean n-space are defined. Two important ones are defined, namely the kernel and the image. To study these further, the idea of linear independence is developed and basis is defined. Having a basis allows us to define the dimension of a subspace. This leads to the major result that for a linear transformation defined on Euclidean n-space, the sum of the dimensions of the kernel and the image are n. Once we have bases, we can define vectors in the subspace in terms of them, giving us coordinates. We then need to study the different matrices a linear transformation gives rise to depending on the choice of basis.

Videos:

3part01.mp4 (7:49)

Lots introduced: linear combinations, span, Image(T). We show that the columns of a matrix span the Image(T).

3part02.mp4 (9:09)

The kernel is defined and we show how to compute it after we develop its properties, such as its connection with rank. Next, we study invertible matrices using rank and row reduced echelon form of the matrix A and connect this with the kernel and the image.

3part03.mp4 (5:36)

Some examples of the kernel and image are discussed.

3part04.mp4 (11:22)

This is a major video. It defines subspaces, linear independence, linear dependence, bases, and develops these concepts.

3part05.mp4 (8:14)

This is a computation based course (there is no understanding if you can't computer), and so in this video we learn how to decide if a set of vectors is linear independent or linearly dependent.

3part06.mp4 (7:59)

A computation to find a basis.

3part07.mp4 (7:49)

We do several simple things here. First, we define the orthogonal complement of a subspace and show it is also a subspace. Then we do a small example of computing the basis for the image of a linear transformation and for the kernel. Then we prove a new property of a linear transformation: Then image of a linearly dependent set is also linearly dependent.

3part08.mp4 (7:52)

In this video a number of basic facts are developed. We show that there is at most n linearly independent vectors in Euclidean n-space. Then we show that if we have linearly independent vectors and another vector not in the span, then adding it to our vectors they are still linearly independent. We show that orthogonal vectors are linearly independent.

3part09.mp4 (10:55)

We define and develop the concept of the dimension of a subspace, non-trivial and fundamental. All bases of a subspace have the same number of vectors.

3part10.mp4 (10:24)

We apply what we just learned to Euclidean n-space and give more properties of a basis.

3part11.mp4 (9:24)

How to find a basis for the kernel (by example).

3part12.mp4 (10:06)

How to find a basis for the image. BIG result: relations between dimension of kernel and image.

THIS IS A GOOD POINT TO BREAK BETWEEN WEEK # 4 AND WEEK # 5.

3part13.mp4 (10:06)

A concrete (but longwinded) example of computing bases for the kernel and image.

3part14.mp4 (4:44)

A short discussion about basis, rank, invertibility, and their connections.

3part15.mp4 (9:25)

Another concrete example. Given a bunch of vectors that span a subspace, find a subset of the vectors that gives a basis.

3part16.mp4 (5:36)

An example is worked where the question has to be reduced to solving equations, but the connection isn't obvious to begin with.

3part17.mp4 (8:18)

An example is worked where vectors are given: are they a basis? Same question with a variable inserted.

3part18.mp4 (6:10)

In IMPORTANT new concept is introduced, namely, the concept of coordinates using a basis. This concept is difficult and easy to confuse. It will plague us for the rest of the semester. The sooner you understand it, the better off you are. This is introduced with a simple example.

3part19.mp4 (12:28)

The simple example of orthogonal projection onto a line in the plane is used to show how useful the above concept of coordinates in a non-standard basis can be.

3part20.mp4 (8:55)

MAJOR VIDEO! We show how a linear transformation has a matrix with respect to any basis, and the matrix is different from the standard matrix using the standard basis, but the linear transformation is the same. We show how to relate the two matrices using different bases. Again this is a place that students often get confused about.

3part21.mp4 (9:00)

We define and study similar matrices (because they arose in the previous video). We compute a previously studied example from this perspective.

3part22.mp4 (7:20)

An example: How to find the matrix for a linear transformation given a new basis.

3part23.mp4 (13:04)

We work two examples, (1) finding the coordinates of a vector in terms of a new basis. (2) Given a nice transformation defined in terms of our new coordinates, what is the matrix in terms of standard coordinates.

Week # 6

Text: Chapter 4 (minus one video). 8 videos, 75 minutes.

Overview:

The notion of abstract linear space (vector space) is introduced and we to for them what we have already done for Euclidean space.

Videos:

4part01.mp4 (8:21)

We define linear space, i.e. vector space. This abstracts Euclidean n-space. Examples are given.

4part02.mp4 (7:36)

More examples.

4part03.mp4 (7:37)

We continue repeating stuff we've already done in this more abstract setting, including invertible, i.e. isomorphic, linear spaces, with some new examples that setting allows for.

4part04.mp4 (6:11)

We show that all finite dimensional abstract linear spaces are isomorphic to Euclidean n-space. Then we work an example of checking to see if some vectors in an abstract space are a basis.

4part05.mp4 (10:07)

We first work a simple example of finding the matrix for a linear transformation given a particular basis in an abstract linear space. Then we show how to compute the matrix for T:V→V given a basis for V. Then we should how to relate this matrix to the matrix we get if we use a different basis to compute a matrix. The diagram at the end is really important.

4part06.mp4 (6:11)

A computational example in our new setting, getting a matrix and a basis for image and kernel.

4part07.mp4 (13:31)

Another exotic example beat to death.

4part08.mp4 (15:23)

Another longwinded example where the basis vectors don't "look right". This requires keeping a clear head and always going back to definitions. Important.

Week # 7

Text: Sections 5.1 and 5.2 (plus one video left from 4.3). 7 videos, 72 minutes.

Overview:

This week focuses on orthogonal vectors and orthogonal subspaces, in particular, we find how to do orthogonal projection and we also learn how to take a basis and construct an orthonormal basis from it.

Videos:

4part09.mp4 (18:16)

Another endless example of the sort you are expected to know how to do. In this case we really work everything using 2 different bases and do the appropriate comparison.

5.1-part1.mp4 (7:09)

We review orthogonality in Euclidean space.

5.1-part2.mp4 (8:29)

We continue the study of orthogonal vectors in Euclidean space, including the concept of the orthogonal complement.

5.1-part3.mp4 (8:58)

Here we study orthogonal projection onto a subspace of Euclidean space and even find formulas for this.

5.1-part4.mp4 (9:47)

The Pythagorean theorem done right (in n-space), and consequences like the Cauchy-Schwartz inequality.

5.1-part5.mp4 (6:57)

Amazing! We can calculate the angle between two vectors in n-space. This also explains the foundation for correlation coefficients.

5.2-part1.mp4 (11:51)

In the previous videos, we often needed to have an orthonormal basis in order to do our computations. In this video we show how to bend a given basis to our will and turn it into and orthonormal basis. This is called the Gram-Schmidt process, and you need to be able to do it. As a bonus, this process gives us a way to factor an arbitrary matrix with linearly independent columns into the product of two very nice matrices. We will see other examples of this kind of analysis later.

Week # 8

Text: Sections 5.3, 5.4, and most of 5.5. 11 videos, 118 minutes.

Overview:

These sections study orthogonal transformations and orthogonal projections, with applications to problems of least squares, line of best fit for a scatter plot, and general data fitting. At the end, inner products on linear spaces are introduced and applied.

Videos:

Section 5.3

01-review-and-goal.mp4 (11:04)

This is review, mainly of 5.1, consolidating what we need for the next few videos.

02-more-review.mp4 (3:56)

Again, more review, this for 5.2.

03-orthogonal-transformations.mp4 (8:42)

Orthogonal transformations. They are rigid, so show up often in certain types of real worlds. Anyway, we study their properties. These special transformations preserves the lengths of vectors.

04-orthogonal-transformations.mp4 (9:08)

We find the special type of matrices associated with orthogonal transformations using an orthonormal basis.

05-orthogonal-transformations.mp4 (9:09)

We define the transpose of a matrix and develop its properties. In the process, we discover that the dimension of the subspace spanned by the rows of a matrix is the same as the dimension of the subspace spanned by the columns, even though they are subspaces of different dimensional Euclidean spaces!

06-orthogonal-projection-matrix.mp4 (12:04)

We compute the inverse of an orthogonal matrix (easy). Then we find the matrix for an orthogonal projection to a line in Euclidean space. We generalize this to finding a matrix (nice, non-computational, theoretical version) for the orthogonal projection to a subspace. But, of course, you have to be able to translate this kind of thing into real numbers.

Section 5.4

This sequence needs an explanation. I was once involved with K-12 education pretty thoroughly, and one of the things that drove us all mad was the education establishment introducing the line of best fit for a scatter plot. This group wouldn't teach multiplication to students unless they understood it completely, but here they were, having students do line of best fit in middle school. The line of best fit is quite sophisticated, so I have a sequence of videos doing somewhat more than the book does. I will label those that are optional. I would like you to watch all of them, but that is just because I am ludicrously proud of the sequence. If you do that, it will take a lot of time. So, they are not all required. The whole sequence of 9 videos is 92 minutes, a bit much.

leastsquares01.mp4 (7:45) OPTIONAL

This starts off using material we have not yet covered, but you might want to come back to it when you have covered it. We will eventually generalize the dot product to more abstract linear spaces, and this video discusses a motivating example.

leastsquares02.mp4 (14:12) OPTIONAL

Discusses least squares from the perspective of the Pythagorean theorem.

leastsquares03.mp4 (7:09) OPTIONAL

This video sets up the problem of the line of best fit for a scatter plot and connects it to least squares via the Pythagorean theorem. It is optional because we do it again later.

leastsquares04.mp4 (4:49) OPTIONAL

We describe how to set up a problem and compute a quadratic approximation to a scatterplot. This clearly generalizes to other cases as well. The book does this, but I'm asking a lot of you already, and, I know you've read the book.

leastsquares05.mp4 (6:59) OPTIONAL

We connect the linear algebra version of least squares to the calculus version.

leastsquares06.mp4 (6:29) OPTIONAL

This describes orthogonal projection if we have an orthonormal basis. Something of a review.

leastsquares07.mp4 (18:01) REQUIRED

This video is the core of 5.4. It does a lot. An example of what it does is to consider a system of equations, Ax=b, that has no solution. The best possible approximation to a solution is the solution to the problem where you take orthogonal projection (solving a least squares problem) of b to in the image of A. This does a lot of what we have already done, but without the advantage of an orthonormal basis. Lots of good formulas and even some good theorems.

leastsquares08.mp4 (21:42) REQUIRED (first 7 minutes anyway)

This really sets up the problem of finding the line of best fit for a scatter plot. The first 7 minutes does all you need to see, although the rest is also in the book, so you should see it one place or the other. This actually shows you how the silly formulas in statistics books come about from linear algebra, orthogonal projection and the Pythagorean theorem.

leastsquares09.mp4 (4:38) OPTIONAL

This goes back and looks at a similar result using calculus and shows how you can do it using linear algebra and what we have developed here.

Section 5.5

5.5-part1.mp4 (5:59)

The dot product in Euclidean space is generalized to an inner product on a linear space. Linear spaces were already a great leap forward in abstraction but this compounds the abstraction with an entirely new level. Survive this and you can do anything.

5.5-part2.mp4 (7:35)

We give an example of one of our new inner products.

5.5-part3.mp4 (9:50)

Our new inner product allows us to talk about the length of a vector and orthogonality, including the concept of an orthonormal basis and orthogonal projection.

Week # 9

Text: The rest of 5.5 and then Sections 6.1 and part of 6.2. 10 videos, 75 minutes.

Overview:

We finish up section 5.5 with a long computation. Then, in sections 6.1 and 6.2, we define determinants and learn how to compute them. The videos give an alternate approach from the book.

Videos:

5.5-part4.mp4 (9:25)

An example of our orthogonal projection in our new situation. We want to give a linear approximation of e to the x on [0,1], but unlike Taylor series, our approximation is best possible for [0,1], not just near a point. Before we can do this, we have to find an orthonormal basis for linear polynomials using our new inner product.

5.5-part5.mp4 (4:59)

We give an alternate way of finding the orthonormal basis in the previous problem.

5.5-part6.mp4 (11:53)

We finally find the linear approximation for e to the x on [0,1] using the results of the previous videos.

det-part1.mp4 (7:08)

We give an alternative approach to determinants, different from that in the book. Still, read the book.

det-part2.mp4 (10:10)

How to compute determinants.

det-part3.mp4 (7:27)

We study determinants from the perspective of the elementary matrices that give the row operations.

det-part4.mp4 (6:14)

We compute some determinants.

det-part5.mp4 (8:01)

We show that the determinant of the product of matrices is the product of the determinants.

det-part6.mp4 (6:28)

We show that the determinant of the transpose of A is the same as the determinant of A.

det-part7.mp4 (4:44)

Compute a determinant with an unknown in the matrix, something we have to get used to.

Week # 10

Text: The last part of 6.2 and 6.3. 7 videos, 50 minutes.

Overview:

We present a good way to compute determinants and then explain what determinants mean geometrically, connecting them up with the chain rule in Calculus III.

Videos:

det-part8.mp4 (3:51)

The determinant of an upper triangular matrices is easy, and reducing the determinant of a matrix to that of an upper triangular matrix is easier than the above approaches. Another way to compute the determinant.

det-part9.mp4 (5:56)

We connect the definition of determinant that we have used to that used by the book.

6.3-part01.mp4 (10:04)

We begin to explain what the determinant really does. To begin with here, we show that if the determinant is 0, then the matrix is not invertible, i.e. has rank < n. Then we show that in the 2x2 case, the determinant computes the area of the parallelogram determined by the column vectors.

6.3-part02.mp4 (7:09)

We generalize the above to n-dimensions. The determinant of A is the n-dimensional volume of the n-dimensional parallelepiped given by the columns. An alternate interpretation is that the determinant of A gives the volume of the image of the n-dimensional unit cube under the linear transformation defined by A.

6.3-part03.mp4 (7:08)

We are interested in the m-dimensional volume defined by m-vectors in n-space where m < n. We use our old factorization of the non-square matrix that we had from the Gram-Schmidt process. However, we do some theory to get a formula that avoids having to do any of that. Neat stuff.

6.3-part04.mp4 (7:00)

You need determinants in Calculus III for the change of variables theorem that allows you to simplify the computation of various multiple integrals. Our interpretation of the determinant as multiplying volumes explains why the determinant shows up in the change of variables formula. We can then go back to Calculus I and see that the change of variables there is the same, it is just that the determinant is of a 1x1 matrix. This is a major reason for doing determinants in this course, to prepare you for Calculus III.

6.3-part05.mp4 (9:09)

We show how we can use determinants to find a nice closed formula for the solution to Ax=b when the determinant of A is non-zero, i.e. when there is a unique solution.

Weeks # 11 and # 12

Text: Chapter 7 (minus Section 7.6). 9 videos, 80 minutes.

Overview:

We introduce a really nice way to analyze a linear transformation through eigenvalues and eigenvectors, at least when it works.

At the end, there is an optional sequence applying what we learn to Fibonacci numbers.

Videos:

7part1.mp4 (9:12)

We define eigenvectors and eigenvalues and begin their study.

7part2.mp4 (9:28)

We do the general theory of how to find eigenvectors and eigenvalues if they exist and then do an explicit computation finding some.

7part3.mp4 (12:29)

We finish up the problem in the previous video and find ourselves with a basis of eigenvectors. We show how much nicer the linear transformation looks if we use this basis, and we connect it with the matrix that originally gave us our linear transformation.

7part4.mp4 (10:03)

Considering the nxn case, we define the characteristic polynomial. We can then talk about algebraic versus geometric multiplicity for eigenvalues.

THIS IS A GOOD POINT TO BREAK BETWEEN WEEK # 11 AND WEEK # 12.

7part5.mp4 (9:17)

We study the ideal case when we have a basis of eigenvectors for $T:V \rightarrow V$, and set up a way to determine if such a basis exists.

7part6.mp4 (5:58)

Recall that similar matrices arise when we study a linear transformation from the perspective of different bases. We now study similar matrices in the context of eigenvalues and eigenvectors.

7part7.mp4 (6:08)

A summary of what we know so far about eigenvectors and eigenvalues.

7part8.mp4 (6:10)

A summary of how to compute such things.

7part9.mp4 (12:20)

We introduce complex numbers for eigenvalues and eigenvectors because we need this later on in the course. We illustrate with an example.

OPTIONAL sequence of videos studying Fibonacci numbers using the techniques from this chapter. The first 2 videos can be safely ignored if you are really interested.

Fibonacci01.mp4 (6:39) DOUBLE OPTIONAL

We define Fibonacci numbers and give a fancy formula for them that looks weird. This can be skipped, even in this sequence.

Fibonacci02.mp4 (5:41) DOUBLE OPTIONAL

We prove the above fancy formula for Fibonacci numbers using induction. This is very unsatisfying because it gives no idea where the formula comes from. This video can also be skipped without missing the crucial lesson.

Fibonacci03.mp4 (7:13) OPTIONAL

This video is necessary for the sequence. Here, we reduce finding a formula to a matrix problem and then further reduce it to an eigenvalue and eigenvector problem.

Fibonacci04.mp4 (10:55) OPTIONAL

Here we do the computation from the previous video setup and show how the truly weird formula comes up naturally from this perspective. Well, not quite. The computation is long and is finished up in the next video.

Fibonacci05.mp4 (9:39) OPTIONAL

Now we see the weird formula coming out in the wash.

Week # 13

Text: Chapter 8. 24 videos, 212 minutes (most of these minutes are working out long computational problems).

Overview:

We continue our analysis of linear transformations using several different approaches. We find when we have a basis of orthonormal eigenvectors. We use this to analyze quadratic functions and associated geometric objects. Then we give a thorough analysis of linear transformations between inner product spaces giving the singular value decomposition theorem. Most of the video time is spent working longwinded examples.

Videos:

8part01.mp4 (5:48)

We ask the question of when A has a basis of orthonormal eigenvectors, the best of all possible worlds.

8part02.mp4 (5:21)

We start on our solution to the above problem beginning with a 2x2 case.

8part03.mp4 (9:49)

We use our complex number results from the previous chapter to make progress on the problem.

8part04.mp4 (10:04)

We finish our solution to the problem of when A has a basis of orthonormal eigenvectors. The answer is surprisingly nice (and complete).

8part05.mp4 (9:18)

We work an example.

8part06.mp4 (10:19)

A bigger more complicated example.

8part07.mp4 (3:04)

An easy way to solve the previous problem by inspection (i.e. cheating).

8part08.mp4 (9:43)

We start solving another ugly problem given a symmetric matrix where we look for a basis of orthonormal eigenvectors.

8part09.mp4 (11:52)

We finish the problem from the previous video, including finding the matrix for the change of basis.

8part10.mp4 (11:46)

We apply this theorem to study quadratic functions, a serious application of the results we have.

8part11.mp4 (10:48)

We use our new understanding to show that the equations for an ellipse must give axes that are perpendicular to each other. There are no weird ellipses.

8part12.mp4 (8:08)

We work an example of an ellipse where we can go from the formula to finding the principle axes for it.

8part13.mp4 (4:31)

We use our new knowledge to compute the points on the above ellipse that are closest and furthest from the origin and find the distances.

8part14.mp4 (6:06)

Looking at a 2-dimensional surface in 3-space defined by a quadratic form we compute the axes and find the closest points to the origin.

8part15.mp4 (8:44)

Another longwinded example of a surface in 3-space, finding the closest points to the origin.

8part16.mp4 (10:43)

A theory video for a change. We give (one version of) a complete analysis of linear transformation between linear spaces with inner products.

8part17.mp4 (11:04)

We take what we just learned and use it to find a decomposition of A, an nxm matrix, into a product of 3 matrices where each of the 3 are very nice. This is the singular value decomposition theorem. You can even compute all this stuff!

8part18.mp4 (4:54)

Just a short geometrical explanation of some of what is going on here.

8part19.mp4 (9:34)

A simple 2x2 computation to get the singular value decomposition. This takes 2 videos even for this simplest of all possible cases.

8part20.mp4 (9:11)

We managed to compute all 3 matrices for the singular value decomposition for our simple 2x2 case.

8part21.mp4 (10:53)

Another example, this time a 3x2 case. It takes 2 videos.

8part22.mp4 (8:18)

We finish our decomposition and give a geometric explanation of what happened.

8part23.mp4 (5:28)

We easily use the previous result to write down the singular value decomposition for the transpose of the matrix there. This new one is a 2x3 matrix.

8part24.mp4 (15:30)

We do the above 2x3 matrix from scratch and check that we get the same answer as before.