problems	1(4)	2(6)	3(4)	4(9)	5(6)	6(8)	7(10)	*total*(47)
scores								

Exam #1, March 2, Linear Algebra (201), Spring, 2018, W. Stephen Wilson

Name : ______________________________

TA Name and section: ______________________________

NO CALCULATORS, NO PAPERS, SHOW WORK .

1. (4 points total, 2 points each) (a) Let $A = (0)$ be the 1×1 matrix for a linear transformation $\mathbb{R} \longrightarrow \mathbb{R}$. Find a basis for the kernel. (b) Let $A = (1)$ be the 1×1 matrix for a linear transformation $\mathbb{R} \longrightarrow \mathbb{R}$. Find a basis for the image.

2. (6 points total, 2 points each) (a) For a linear transformation, $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$, using the standard basis $\{\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n\}$, what is the k-th column of the matrix A for T? (b) Using the basis $\mathscr{B} = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$, what is the k-th column of the matrix B for T? (c) Let S be the matrix with columns $\vec{v}_k$. What is the relationship between the matrices A and B in parts (1) and (2) above?

3. (4 points, 2 points each) Two true false questions. 2 points for the correct answer, 1 if you leave it blank, and 0 if you get it wrong. (a) There exists a 3×3 matrix with $\ker A = \text{image } A$. (b) If $\vec{v}_1, \ldots, \vec{v}_n$ are linearly independent vectors and T is a linear transformation, then $T(\vec{v}_1), \ldots, T(\vec{v}_n)$ are linearly independent vectors.

4. (9 points, 3 points each) Let the linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be given by the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ with respect to the standard basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$. Take a new basis by reordering these so we have $\mathscr{B} = \{\vec{e}_3, \vec{e}_1, \vec{e}_2\}$. (a) Find the matrix S so that $\vec{x} = S\vec{x}_{\mathscr{B}}$. (b) Find the inverse of the matrix S, S^{-1}. (c) Find the matrix B of the linear transformation T with respect to the new basis $\mathscr{B}$.

5. (6 points, 3 points each) Let the linear transformation $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^5$ be given by the matrix

$$A = \begin{pmatrix} 1 & -1 & 1 & -2 \\ 2 & -2 & 1 & -3 \\ 1 & -1 & 2 & -3 \\ 1 & -1 & 0 & -1 \\ 3 & -3 & 1 & -4 \end{pmatrix}.$$

(a) Find a basis for the kernel of A. (b) Find a basis for the image of A.

The next two problems are the *hard* problems. They can be worked in either order as each could help with the other. After working the exam myself, I feel the last problem is easiest and has more points. There is a blank page after each problem. Don't miss the last problem.

6. (8 points, 2 for each entry in the matrix) Consider the basis $\mathscr{B} = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$ for a subspace $V \subset \mathbb{R}^3$. Using $\mathscr{B}$ for your basis, find the 2×2 matrix B for the linear transformation $V \longrightarrow V$ that is the orthogonal projection of V to the line, L, in V spanned by $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

Black page for work. There is another problem on the next page.

7. (10 points, 2 for each entry in the coordinates) Consider the basis $\mathscr{B} = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$ for a subspace $V \subset \mathbb{R}^3$. Find a non-zero vector $\vec{x} \in V$ that is perpendicular to $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$. Give your answer in two forms: (a) coordinates in the standard basis and (b) coordinates in the basis $\mathscr{B}$.

This blank page is for work. End of exam.